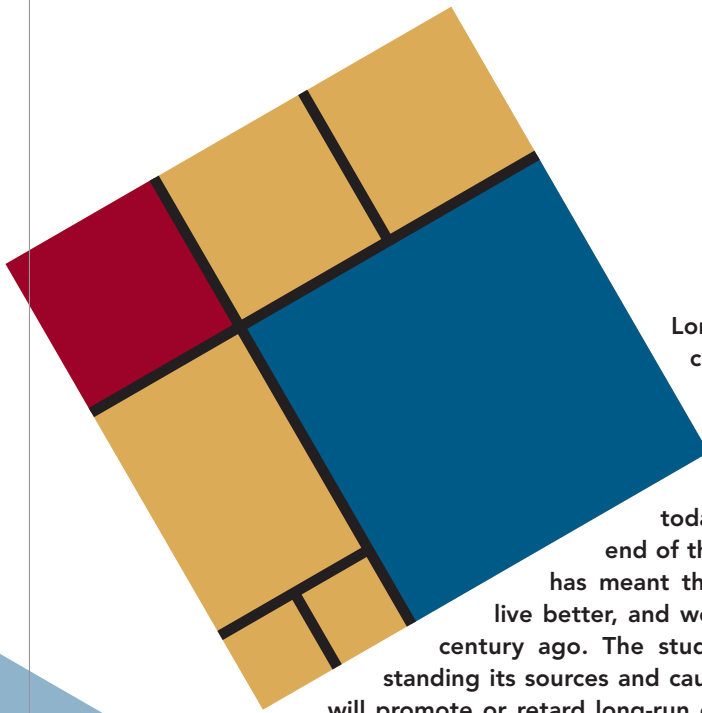


# Long-Run Economic Growth

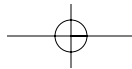
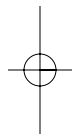
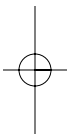
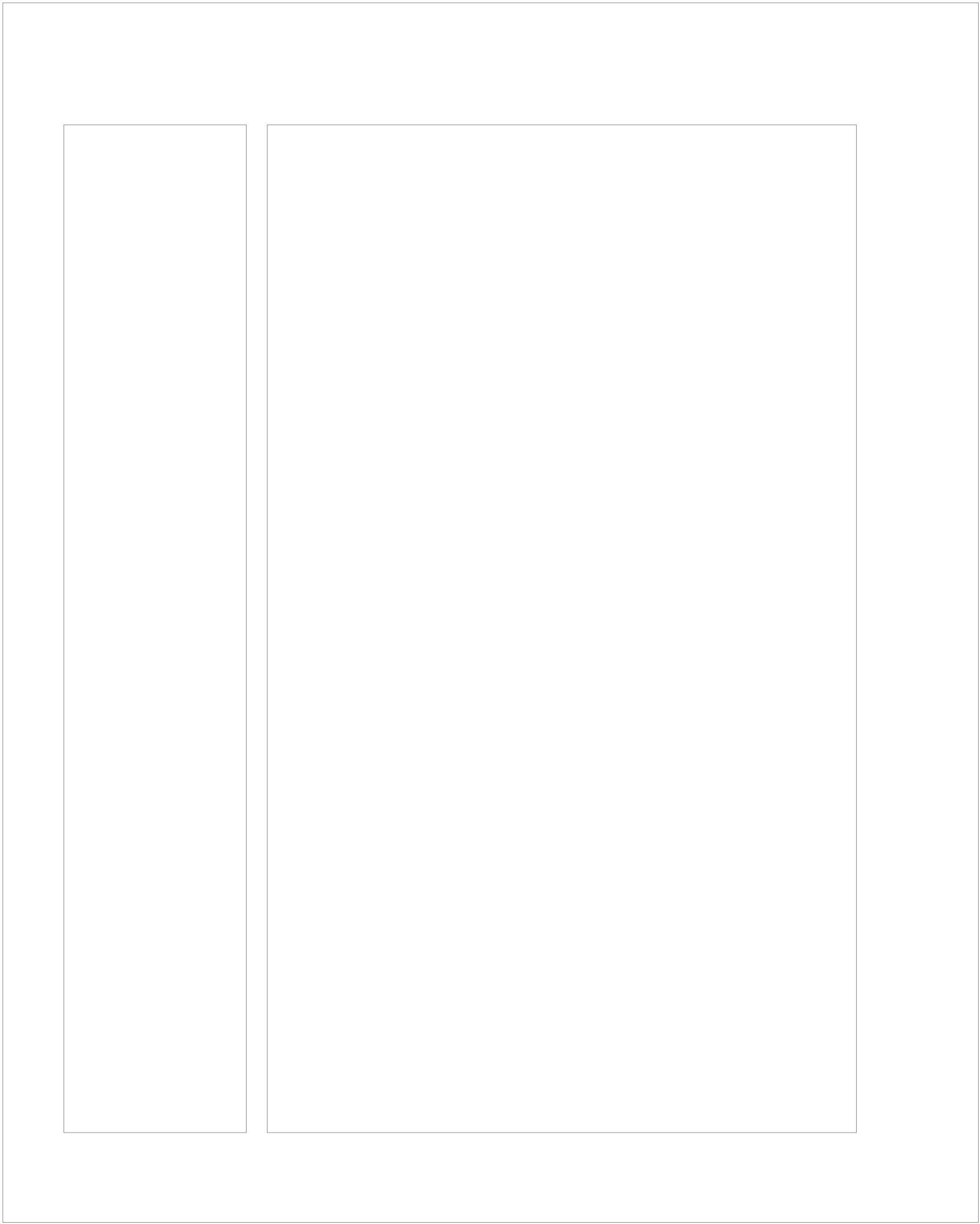
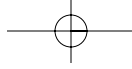
## 2 PART



Long-run economic growth is the subject of this two-chapter part of the book. Chapter 4 covers the theory of growth, and Chapter 5 covers the worldwide pattern of economic growth. Long-run economic growth is the most important topic in macroeconomics. Standards of living in the United States today are at least seven times what they were at the end of the nineteenth century. Successful economic growth has meant that almost all citizens of the United States today live better, and we hope happier, lives than even the rich elite of a century ago. The study of long-run economic growth aims at understanding its sources and causes and at determining what government policies will promote or retard long-run economic growth.

Long-run growth is a module that is not very closely connected to the study of business cycles, recessions, unemployment, inflation, and stabilization policy that make up the bulk of this book. The models used and the conclusions reached in Part 2 will by and large not be central to subsequent parts for starting in Chapter 6 we turn to business cycles.

Why, then, include this two-chapter part? The principal reason is that long-run economic growth is such an important topic.



CHAPTER  
4

## The Theory of Economic Growth

### QUESTIONS

What are the causes of long-run economic growth — that is, of sustained and significant growth in an economy's level of output per worker?

What is the "efficiency of labor"?

What is an economy's "capital intensity"?

What is an economy's "balanced-growth path"?

How important is faster labor-force growth as a drag on economic growth?

How important is a high saving rate as a cause of economic growth?

How important is technological and organizational progress for economic growth?

## 4.1 SOURCES OF LONG-RUN GROWTH

Step back and take a broad, sweeping view of the economy. Look at it, but do not focus on the “short run” of calendar-year quarters or even of a year or two in which shifts in investment spending and other shocks push the unemployment rate up or down — that’s what we will do in Chapters 9 through 12. Look at it, but do not focus on the “long run” period of 3 to 10 years or so, in which prices have time to adjust to return the economy to a full-employment equilibrium but in which the economy’s productive resources do not change much — that’s what we will look at in Chapters 6 through 8. What do we do here in Chapters 4 and 5? We take that step back and focus on the very long run of decades and generations — a period over which everything else dwindles into insignificance except the sustained and significant increases in standards of living that we call long-run economic growth.

When we take this broad, sweeping view, it is clear that what we are calling economic growth is the only truly important factor. As Table 4.1 shows, material

### economic growth

The process by which productivity, living standards, and output increase.

**TABLE 4.1**

Current GDP per Capita Levels for Countries with More than 50 Million People

The World Bank’s latest estimates of GDP per capita levels, measured at purchasing power parities. (That is, currencies are converted into dollars at a rate that gives approximately the same purchasing power both before conversion in the other country and after conversion in the United States.) The final column shows how far the country’s GDP per capita is “behind” the United States — when the United States’s level of GDP per capita was last below that of the country’s current value.

Country	GDP per Capita	Matches U.S. Level in . . .
United States	\$35,060	2004
Germany	26,220	1988
France	26,180	1988
Japan	26,070	1988
United Kingdom	25,870	1987
Italy	25,320	1987
Mexico	8,540	1940
Russia	7,820	1940
Brazil	7,250	1939
Thailand	6,680	1928
Iran	6,340	1925
Turkey	6,120	1924
China	4,390	1900
Philippines	4,380	1900
Egypt	3,710	1897
Indonesia	2,990	1879
India	2,570	1874
Vietnam	2,240	1854
Pakistan	1,940	1849
Bangladesh	1,720	1836
Nigeria	780	—
Ethiopia	720	—
Congo	580	—

Source: World Bank

standards of living and levels of economic productivity today in the United States are more than four times what they are in, say, Mexico (and more than nine times those of Egypt, and more than 40 times those of Nigeria). Only a trivial part of these differences is due to whether unemployment in a country is currently above or below its average level or whether various bad macroeconomic policies are currently disrupting the functioning of the price system. The overwhelming bulk of these differences is the result of differences in economies' productive potentials and in the factors that determine productive potential — the skills of the **labor force**, the value of the **capital stock**, and the level of technology and organization currently used in production.

These enormous gaps between the productive potentials of different nations spring from favorable initial conditions and successful growth-promoting economic policies in the United States — and from less favorable initial conditions and less successful policies in Mexico and downright unsuccessful policies in Egypt and Nigeria. As Figure 4.1 shows, material standards of living and levels of economic productivity in the United States today are at least seven times what they were at the end of the nineteenth century (and more than 30 times what they were at the founding of the republic). The bulk of today's gap between living standards and productivity levels in the United States and Mexico (and Egypt

**labor force**

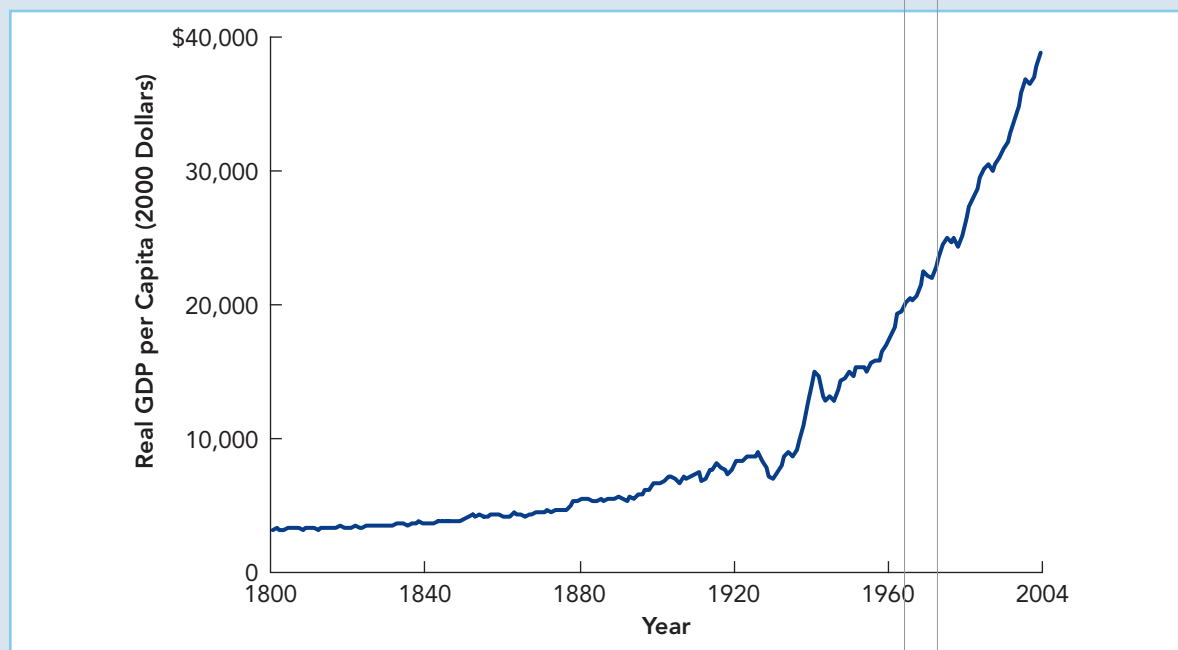
The sum of those who are employed and those who are actively looking for work.

**capital stock**

The economy's total accumulated stock of buildings, roads, other infrastructure, machines, and inventories.

**FIGURE 4.1**

**American Real GDP per Capita, 1800–2004 (in 2000 Dollars)** The pace of modern economic growth in the United States has been astonishing. Economic historians estimate — and if anything, their estimates are underestimates — economic product per person in the United States measured in 2000 prices has grown from \$1,200 at the time of the writing of the Constitution to over \$36,000 in 2004.



Source: Louis Johnston and Samuel H. Williamson (2004), "The Annual Real and Nominal GDP for the United States, 1789–Present." Economic History Services, March 2004, <http://www.eh.net/hmit/gdp/>.

and Nigeria) opened up in the past century; the bulk of success (or failure) at boosting an economy's productive potential is thus — to a historian at least — of relatively recent origin.

Successful economic growth means that nearly all citizens of the United States today live better — along almost every dimension of material life — than did even the rich elites of preindustrial times. If good policies and good circumstances accelerate economic growth, bad policies and bad circumstances cripple long-run economic growth. Argentines were richer than Swedes before World War I, but Swedes today have four times the standard of living and the productivity level of Argentines.

We classify the factors that generate differences in economies' productive potentials into two broad groups:

- First, differences in the economy's *efficiency of labor* — how technology is deployed and organization is used to increase the amount of output a worker can produce, even with the same amount of capital.
- Second, differences in the economy's *capital intensity* — how large a multiple of current production has been set aside in the form of useful machines, buildings, and infrastructure to boost the productivity of workers, even with the same technology or organization.

Economists have spent much time over the past two generations dividing economic growth into that part due to improvements in technology and in social and business organization that boost the efficiency of labor, on the one hand, and into that part generated by investment in capital to boost the economy's capital intensity — the ratio of the capital stock to output — on the other. A very important finding is that investment that boosts capital intensity plays a substantial role in generating growth. But an even more important finding is that the lion's share of economic growth comes from factors that affect the efficiency of labor.

In this two-chapter section on economic growth, this first chapter, Chapter 4, sets out economists' basic theory of economic growth. It presents the concepts and models economists use to organize their thinking about the causes of long-run economic growth and stagnation. Chapter 5 then sketches the facts of economic growth over time and across the world. Chapter 4 is a tour of how economists think about economic growth. Chapter 5 is a tour of how economic growth is progressing or not progressing around the world.

Now let's begin our tour of how economists think by looking in more detail at the two broad groups into which economists divide factors affecting long-run economic growth: the efficiency of labor, on the one hand, and capital intensity, on the other.

### The Efficiency of Labor

The biggest reason that Americans today are vastly richer and more productive than their predecessors of a century ago is that they have enjoyed an extraordinary amplification of the **efficiency of labor**. Efficiency has risen for two reasons: advances in technology and in organization. We now know how to make electric motors, dope semiconductors, transmit signals over fiber optics, fly jet airplanes, machine internal combustion engines, build tall and durable structures out of concrete and steel, record entertainment programs on DVDs, make hybrid seeds, fertilize crops with nutrients, organize assembly lines, and a host of other things our predecessors did not know how to do. These technological advances allow American workers to

#### efficiency of labor

The skills and education of the labor force, the ability of the labor force to handle modern technologies, and the efficiency with which the economy's businesses and markets function.

easily perform value-generating tasks that were unimaginable, or maybe only extremely difficult to accomplish, a century ago.

Moreover, the American economy is equipped to make use of all these technological capabilities. It has the forms of business organization, it has the stability and honesty of government, it has the schools to educate its population, and it has the other socioeconomic institutions needed to successfully utilize modern technology. So it is both better technology and advances in organization that have led to vast increases in the efficiency of labor and thus to American standards of living.

Thus admitting that we economists know less than we should about the processes by which this better technology and these advances in organization come to be is somewhat awkward. Economists are good at analyzing the consequences of advances in technology and improvements in other factors that make for a high efficiency of labor, but they have less to say than they should about their sources.

### Capital Intensity

A secondary but still large part of America's very long run economic growth — and a secondary but still large component of differences in material standards of living across countries today — has been generated by the second source of growth: **capital intensity**. Does the economy have a low ratio of capital per unit of output, that is, relatively little in the way of machines, buildings, roads, bridges, and so on? Then it is likely to be poor. The higher is the economy's capital intensity, the more prosperous the economy will be: A more capital-intensive economy will be a richer and a more productive economy.

#### capital intensity

The ratio of the capital stock to total potential output,  $K/Y$ , which describes the extent to which capital, as opposed to labor, is used to produce goods and services.

### Fitting the Two Together

The task of the rest of Chapter 4 is to build economists' standard model of long-run economic growth, a model into which we can fit these two broad groups of factors. This standard model is called the *Solow growth model*, after Nobel Prize-winning MIT economist Robert Solow. The Solow growth model is a dynamic model of the economy: It describes how the economy changes and grows over time as saving and investment, labor force growth, and progress in advancing technology and improving social organization raise the economy's level of output per worker and thus its material standard of living. Saving and investment are the drivers leading to increases in capital intensity. Progress in technology and organization are the drivers leading to increases in the efficiency of labor.

#### RECAP SOURCES OF LONG RUN GROWTH

In the very long run, economic growth is the most important aspect of economic performance. Two major factors determine economic growth: growth in the efficiency of labor — a product of advances in technology on the one hand and improvements in economic and social organization on the other — and the economy's capital intensity. Policies that accelerate innovation or improve institutions and so boost the efficiency of labor accelerate economic growth and create prosperity, as do policies that boost investment and raise the economy's capital intensity to a higher level.

## 4.2 THE SOLOW GROWTH MODEL

As is the case for all economic models, the Solow growth model consists of

- *Variables*: economic quantities of interest that we can measure.
- *Behavioral relationships*: relationships that (1) describe how humans, making economic decisions given their opportunities and opportunity costs, decide what to do and (2) determine the values of the variables.
- *Equilibrium conditions*: conditions that tell us when the economy is in a position of balance, when the variables we are focusing on are “stable” — that is, when the variables are changing in simple and predictable ways.

Almost every economic model has a single key economic variable at its heart. We will be most interested in that one variable — the one the model is organized around.

In the case of the Solow growth model, the key variable is *labor productivity*: output per worker, how much the average worker in the economy is able to produce. We calculate output per worker by simply taking the economy’s level of real GDP or output  $Y$ , and dividing it by the economy’s labor force  $L$ . This quantity, output per worker,  $Y/L$ , is our best simple proxy for the standard of living and level of prosperity of the economy.

In every economic model — and the Solow growth model is no exception — economists analyze the model by looking for an *equilibrium*: a point of balance, a condition of rest, a state of the system toward which the model will converge over time. Economists look for equilibrium for a simple reason: either an economy is at its (or one of its) equilibrium position, or it is moving — and probably moving rapidly — to an equilibrium position.

Once you have found the equilibrium position toward which the economy tends to move, you can use it to understand how the model will behave. If you have built the right model, it will tell you in broad strokes how the economy will behave. In economic growth, the equilibrium economists look for is an equilibrium in which economy’s capital stock per worker, its level of real GDP per worker, and its efficiency of labor are all three growing at the exact same proportional rate.

The equilibrium economists look for in the case of the Solow growth model is a *balanced-growth equilibrium*. In this growth equilibrium the capital intensity of the economy — its capital stock divided by its total output,  $K/Y$  — remains constant as the rest of the variables in the economy grow. The amount of capital that the economy uses to produce each unit of output remains constant over time, as both the capital stock and output grow at the same proportional rate, and thus capital intensity does not change.

For each balanced-growth equilibrium, there is a **balanced-growth path**: this is a growing economy and so the economy’s variables of interest change over time. We need to know how fast it is growing: at what rate output per worker,  $Y/L$ , is increasing. So, after finding the balanced-growth equilibrium, we calculate this balanced-growth path. Forecasting then becomes straightforward. If the economy is on its balanced-growth path, the present value of output per worker is on and future values of output per worker will continue to follow the balanced-growth path. If the economy is not yet on its balanced-growth path, it will head toward it: over time, the economy will converge to its balanced-growth path.

We will see how this works later on. First we need to set out the pieces of the Solow growth model.

### balanced-growth path

The path toward which total output per worker tends to converge as the capital-output ratio converges to its equilibrium value.

### The Production Function

Let's start with the average worker, a worker whose productivity is simply  $Y/L$ , the economywide average. This average worker uses the economy's current level of technology and organization. These are captured by the current value of the efficiency of labor  $E$ . This average worker also uses an average share of the economy's capital stock: He or she has  $K/L$  worth of capital to amplify his or her productivity. We want to analyze how these two — efficiency of labor  $E$  and the capital-to-labor ratio  $K/L$  — affect the average worker's productivity  $Y/L$ .

To do this, we write down a behavioral relationship that tells us how the average worker's productivity  $Y/L$  is related to the efficiency of labor  $E$  and the amount of capital  $K/L$  at the average worker's disposal. We give this behavioral relationship a name: the **production function**. Tell the production function what resources the economy's average worker has available, and it will tell you how much output the typical worker can produce. In an abstract form we write the production function as

$$\frac{Y}{L} = F\left(\frac{K}{L}, E\right)$$

This says just that there is a systematic relationship between **output per worker**  $Y/L$  — real GDP divided by the number of workers — and the economy's available resources: the capital stock per worker  $K/L$  and the efficiency of labor  $E$ . The pattern of this relationship is prescribed by the form of the function  $F(\ )$ .

As long as all we know about the production function is that we write it with the symbols  $F(\ )$  — one capital letter and one set of parentheses — it is not of much use. We know that there is a relationship between resources and production, but we don't know what that relationship is. We cannot calculate much of anything; we cannot give quantitative answers to any questions we are asked about the effects of changes in economic policy and the economic environment on economic growth.

To make our model more useful, we will give a simple algebraic form to our production function: the Cobb-Douglas form, which we choose primarily because using it makes lots of formulas later in the chapter simpler than they would otherwise be. We write the Cobb-Douglas production function as

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha (E)^{1-\alpha}$$

where  $\alpha$  is a parameter between zero and 1. The economy's level of output per worker  $Y/L$  is equal to the capital stock per worker  $K/L$  raised to the power of a parameter  $\alpha$  (which is a number between zero and 1), and then multiplied by the current efficiency of labor  $E$  itself raised to the power  $(1 - \alpha)$ .<sup>1</sup>

The Cobb-Douglas production function contains a parameter  $\alpha$ . The value of  $\alpha$  tells us how rapidly the economic usefulness of additional *investment* in buildings and machines declines as the economy accumulates more and more of them. That is,  $\alpha$  measures how fast diminishing returns to investment set in, in the economy. A value of  $\alpha$  near zero means that the extra amount of output made possible by an additional unit of capital declines very quickly as the capital stock rises. A value

<sup>1</sup>When we are interested not in the level of output per worker but in the total level of output  $Y$  in the economy, we simply multiply both the left- and right-hand sides of the equation above by the labor force  $L$  to get the total output form of the production function  $Y = (K)^\alpha (LE)^{1-\alpha}$ .

#### production function

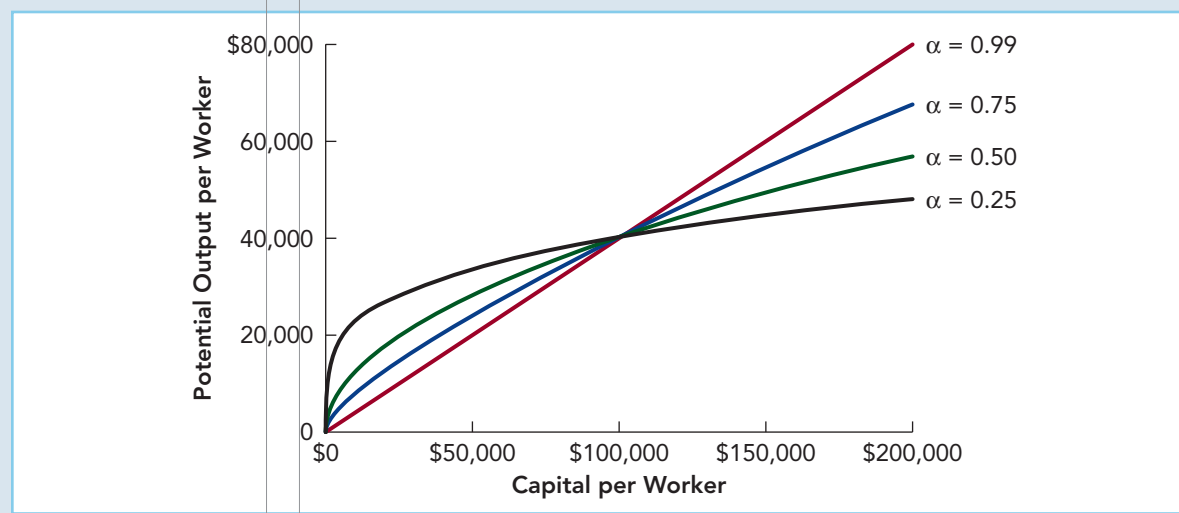
The relationship between the total amount of national product produced and the resources used to produce it: the quantity of capital, the quantity of labor, and the levels of technology and organization that determine the efficiency of labor.

#### output per worker

The average amount of output produced in a year per worker,  $Y/L$ . Equal to the average labor productivity. Used as a proxy for the material standard of living.

**FIGURE 4.2**

The Cobb-Douglas Production Function for Different Values of  $\alpha$  As the exponent  $\alpha$  in the Cobb-Douglas production function changes, the speed with which diminishing returns to investment set in — and thus the curvature of the production function — changes. With a high value of  $\alpha$  near 1, output per worker  $Y/L$  increases nearly one-for-one with the capital stock per worker  $K/L$ . With a low value of  $\alpha$  near zero, the economy quickly reaches the point where additional capital accumulation raises output by only a little. For each of the values of  $\alpha$  plotted in this figure, the value of the efficiency of labor  $E$  has been adjusted so that output per worker  $Y/L$  is equal to \$40,000 a year when the capital stock per worker  $K/L$  is equal to \$100,000.

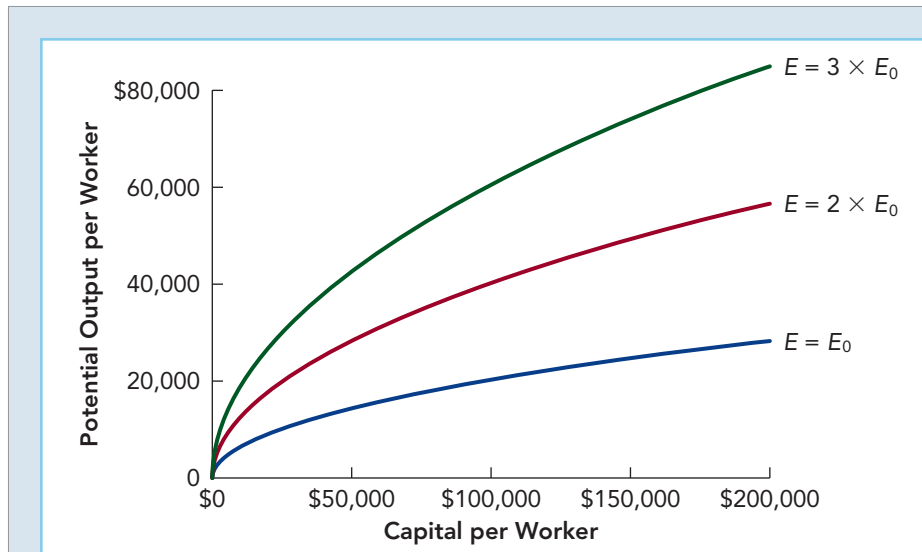


of  $\alpha$  near 1 means that each additional unit of capital makes possible almost as large an increase in output as the last additional unit. As  $\alpha$  varies from a high number near 1 to a low number near 0, the force of diminishing returns to investment gets stronger, as Figure 4.2 shows.<sup>2</sup>

The value of  $\alpha$  determines how quickly the Cobb-Douglas production function flattens out when output per worker is plotted on the vertical axis and capital per worker is plotted on the horizontal axis. The value of the efficiency of labor  $E$  tells us how high the production function rises: A higher level of  $E$  means that more output per worker is produced for each possible value of the capital stock per worker, as Figure 4.3 shows.

The Cobb-Douglas production function is flexible. It can be “tuned” to fit any of a wide variety of different economic situations. Are we studying an economy in which productivity is high? Use the Cobb-Douglas production function with a high value of the efficiency of labor  $E$ , as Figure 4.3 shows. Does the economy rapidly hit the wall as investment proceeds, with little increase in the level of production? Use the Cobb-Douglas function with a low value — near zero — of  $\alpha$ , as Figure 4.2 shows. Is the speed with which diminishing returns

<sup>2</sup>One way of illustrating this point in algebra is to use a little calculus to calculate the *marginal product of capital*, the  $MPK$  — how much total output increases as a result of a one-unit increase in the capital stock. For the Cobb-Douglas production function,  $MPK = \alpha/(K/Y)$ . The higher the current capital-to-output ratio, the lower is the marginal product of capital. And the lower is the parameter  $\alpha$ , the lower is the marginal product of capital.

**FIGURE 4.3****The Cobb-Douglas Production Function for Different Values of  $E$** 

As the parameter  $E$  in the Cobb-Douglas production function increases, the curve representing the function moves upward. With a higher value of  $E$ , each possible value of capital per worker produces a larger value for output per worker.

to investment set in moderate? Pick a middle value of  $\alpha$ . The Cobb-Douglas function will once again fit.

Given a value for the diminishing-returns-to-investment parameter  $\alpha$ , for the efficiency of labor  $E$ , the economy's capital stock  $K$ , and the labor force  $L$ , we can calculate the level of output per worker  $Y/L$  in the economy. Box 4.1 shows how to use the production function.

**USING THE PRODUCTION FUNCTION: AN EXAMPLE**

Suppose we know that the current value of the efficiency of labor  $E$  is \$10,000 a year and that the diminishing-returns-to-investment parameter  $\alpha$  is 0.3. Then determining how the level of output per worker  $Y/L$  depends on the capital stock per worker  $K/L$  is straightforward.

Let's start with the case in which the capital stock per worker is \$125,000. The Cobb-Douglas production function is

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} (E)^{1-\alpha}$$

Substitute in the known values of  $K/L$ ,  $E$ , and  $\alpha$  to get

$$\frac{Y}{L} = (125,000)^{0.3} (10,000)^{0.7}$$

Use your calculator to evaluate the effect of raising numbers to these exponents to get

$$\frac{Y}{L} = (33.812)(630.957)$$

And then multiply to get

$$\frac{Y}{L} = \$21,334 \text{ per year}$$



If we are interested in a capital stock per worker level of \$250,000, the calculations are

$$\frac{Y}{L} = (250,000)^{0.3}(10,000)^{0.7}$$


And, using your calculator,

$$\frac{Y}{L} = (41.628)(630.957)$$

$$\frac{Y}{L} = \$26,265 \text{ per year}$$

Note that the first \$125,000 of capital per worker boosted output per worker from \$0 to \$21,334, while the second \$125,000 of capital per worker boosted output per worker only from \$21,334 to \$26,265 — by less than one-quarter as much. These substantial diminishing returns to investment should come as no surprise: The value of  $\alpha$  is quite low at 0.3, and low values of  $\alpha$  produce rapidly diminishing returns to capital accumulation.

This example offers another important lesson: Keep your calculator or computer's spreadsheet handy! Nobody expects anyone to raise \$250,000 to the 0.3 power in her or his head and come up with 41.628. The Cobb-Douglas form of the production function, with its fractional exponents, carries the drawback that students (or professors) can do problems in their heads or with just pencil and paper only if the problems have been carefully rigged beforehand.

Nevertheless, we use the Cobb-Douglas production function because of its extraordinary convenience: By varying just two parameters we can fit the model to an enormous variety of potential economic situations. 

### Saving, Investment, and Capital Accumulation

In Chapter 6 we will talk in detail about the circular-flow relationship you learned in your Principles of Economics class: the amount of output an economy produces (real GDP, or  $Y$ ) equals total spending, with total spending divided into four parts, consumption spending  $C$ , investment spending  $I$ , government purchases  $G$ , and net exports  $NX$ , which equal gross exports minus imports,  $NX = GX - IM$ .

Here we want to use this relationship to understand how investment spending  $I$  is equal to *total saving*. The answer is that they are very closely related: The net flow of saving — household saving, plus *foreign saving* invested in our country, minus the *government's budget deficit* — is equal to the amount of investment. In the end, there is no place that net saving can go but to finance investment. If you save it, and it gets into the hands of a bank, then unless some other household borrows it (and so offsets your positive saving with its negative saving) the bank will lend it out, and it will be spent on business investment.

To see this more formally, a little bit of easy algebra is all we need. Start from the national income identity: Consumption  $C$  plus investment  $I$  plus government purchases  $G$  plus net exports  $GX - IM$  equals real GDP  $Y$ :

$$C + I + G + GX - IM = Y$$

First subtract *net taxes*  $T$  from both sides:

$$C + I + (G - T) + GX - IM = Y - T$$

Next subtract consumption spending  $C$ , the government budget deficit  $G - T$ , and net exports  $GX - IM$ , from both sides:

$$I = (Y - T - C) - (G - T) + (IM - GX)$$

The right-hand side is the three pieces of total saving: household saving, government saving, and foreign saving.  $Y - T - C$ , real GDP minus net taxes minus consumption spending, is household saving. Call it  $S^H$ ,  $S$  for saving and  $H$  for household.

When government purchases are less than net taxes, the government is running a *surplus* and is saving. Call this  $S^G$ ,  $S$  for saving and  $G$  for government. But with the exception of a few years at the end of the 1990s, the U.S. government has run *deficits*. So to keep our example realistic, we focus here on the government's "dissaving" — borrowing by running a deficit.  $G - T$ , government purchases minus net taxes, is the government's deficit. When the government runs a deficit, its saving  $S^G$  is negative.

$IM - GX$  is simply the excess of imports over exports, which equals the net flow of saving that foreigners invest here. Call it  $S^F$ ,  $S$  for saving and  $F$  for foreign. With these new symbols, our equation above tells us that investment is equal to

$$I = S^H + S^G + S^F$$

The right-hand term,  $S^H + S^G + S^F$ , is the total saving flowing into the economy. The equation says that businesses take all this saving and use it to invest — to buy and install the machines and build the buildings that make up our capital stock.<sup>3</sup>

Now, let's assume that total saving  $S^H + S^G + S^F$  is a constant fraction  $s$  of real GDP  $Y$ :

$$s = \frac{S^H + S^G + S^F}{Y}$$

In other words,  $sY = S^H + S^G + S^F$ . Thus investment spending  $I$  is equal to saving  $sY$ :

$$I = S^H + S^G + S^F = sY$$

In this chapter we will assume that  $s$  is almost always constant. We may think about the consequences of its taking an upward or downward jump or two at some particular moment of time, but the background assumption (made because it makes formulas much simpler) will be that  $s$  will remain at its current value as far as we look into the future. We will call  $s$  the economy's **saving rate** or, more completely, its *saving-investment rate* to remind us that  $s$  is measuring both the flow of saving into the economy's financial markets and also the share of total production that is invested and used to build up and increase the economy's capital stock.

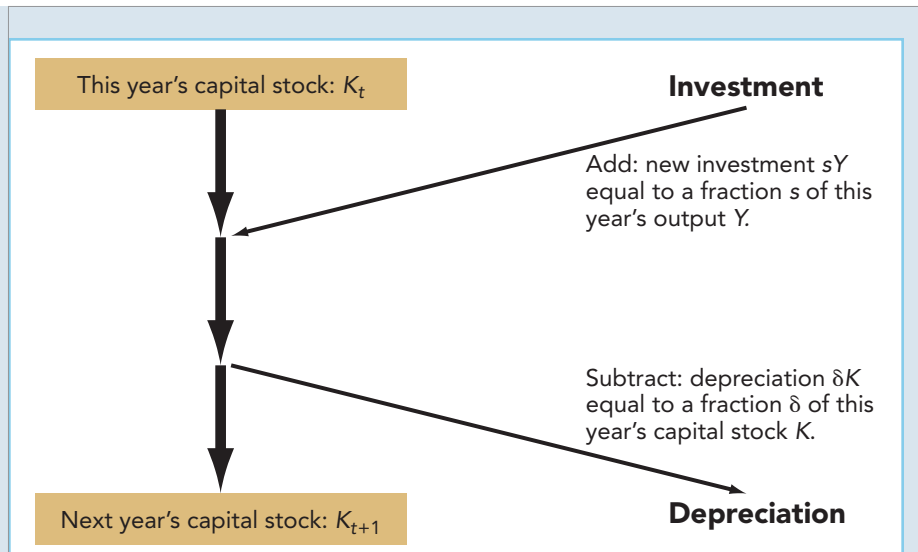
#### saving rate

The share of total GDP that an economy saves,  $s$ , equal to the sum of household, government, and foreign saving divided by total output.

<sup>3</sup>With more time, pages, and patience, we would develop this relationship further, by taking into account the differences between the government's investments that build up publicly owned capital such as roads and bridges, and the remainder of government purchases. But here we will follow economists' custom of ruthless simplification, and say that all investment is captured in  $I$ .

**FIGURE 4.4****Investment and Depreciation**

Gross investment adds to and depreciation subtracts from the capital stock.

**depreciation**

The difference between gross and net investment in capital; the amount by which capital stock wears out, becomes obsolete, or is scrapped over a year.

**capital-output ratio**

The economy's capital stock divided by potential output,  $K/Y$ .

Whereas we assume the saving-investment rate  $s$  is constant, we can't make the same assumption about the capital stock. The economy's capital stock  $K$  is not constant. It changes from year to year. Let's adopt the convention of using a subscript when we need to identify the year to which we are referring.  $K_0$  denotes the capital stock at some initial year, usually the year at which we begin the analysis;  $K_{2003}$  means the capital stock in 2003;  $K_t$  denotes the capital stock in the current year;  $K_{t+1}$  means the capital stock next year; and  $K_{t-1}$  means the capital stock last year.

Over time investment makes the capital stock tend to grow. But over time **depreciation** makes the capital stock tend to shrink — old capital becomes obsolete, or breaks, or simply wears out. We make a simple assumption for depreciation: The amount of capital that wears out, breaks, and becomes obsolete in any year is simply a constant parameter  $\delta$  (lowercase Greek delta, the depreciation rate) times the current capital stock. Thus we can write that next year's capital stock will be<sup>4</sup>

$$K_{t+1} = K_t + \text{Investment} - \text{Depreciation}$$

$$K_{t+1} = K_t + sY_t - \delta K_t$$

as illustrated in Figure 4.4.

From these definitions, we see clearly that the capital stock is constant — that this year's capital stock is equal to next year's — when

$$sY_t = \delta K_t$$

And this is true when the **capital-output ratio** is

$$\frac{K_t}{Y_t} = \frac{s}{\delta}$$

<sup>4</sup>Sometimes you will hear people refer to the change in the capital stock as *net investment* — investment net of depreciation — and refer to the amount of plant and equipment purchased and installed as *gross investment*. In this book “investment” will mean *gross investment*, and we will use “investment minus depreciation” in place of *net investment*.

Now suppose for a page or two that our economy has no labor force growth and no growth in the efficiency of labor. (We will immediately drop this assumption when we leave this subsection, and we include it here only because understanding a more complex case later will be easier if we understand the simpler case now.) The labor force is constant at its initial value  $L_0$ . The efficiency of labor is constant at its value  $E_0$ .

If the capital-output ratio  $K/Y$  is lower than  $s/\delta$ , then depreciation ( $\delta K$ ) is less than investment ( $sY$ ) so the capital stock and the capital-output ratio will grow. They will keep growing until  $K/Y$  reaches  $s/\delta$ . If the capital-output ratio  $K/Y$  is greater than  $s/\delta$ , then depreciation ( $\delta K$ ) is more than investment ( $sY$ ), so the capital stock and the capital-output ratio will shrink. They will keep shrinking until  $K/Y$  falls to  $s/\delta$ . In the very long run, the capital-output ratio will be  $s/\delta$ . The requirement

$$\frac{K_t}{Y_t} = \frac{s}{\delta}$$

is the equilibrium condition in this particular simple case of the Solow growth model — the case in which there is neither growth in the labor force nor growth in the efficiency of labor.

What is that equilibrium value of output per worker? A little algebra detailed in Box 4.2 shows us that we can move back and forth between the capital-per-worker

#### THE TWO FORMS OF THE PRODUCTION FUNCTION: SOME DETAILS

How do we transform our production function from one that focuses on the capital-labor ratio to one that focuses on the capital-output ratio? Through a little bit of simple algebra. Begin with our capital-per-worker form of the Cobb-Douglas production function:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha (E)^{1-\alpha}$$

Rewrite  $(K/L)$  as  $(K/Y)(Y/L)$ :

$$\frac{Y}{L} = \left(\frac{K}{Y} \frac{Y}{L}\right)^\alpha (E)^{1-\alpha}$$

Regroup:


$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^\alpha \left(\frac{Y}{L}\right)^\alpha (E)^{1-\alpha}$$

Divide both sides by  $(Y/L)^\alpha$ :

$$\left(\frac{Y}{L}\right)^{1-\alpha} = \left(\frac{K}{Y}\right)^\alpha (E)^{1-\alpha}$$

Finally, raise both sides to the  $1/(1-\alpha)$  power in order to get  $Y/L$  by itself on the left-hand side:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} (E)$$

This is the capital-output ratio form of the production function. 



form of the production function that we have already seen,

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha (E)^{1-\alpha}$$

and a different (but more convenient) form, the capital-output ratio form of the production function:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} (E)$$

Both forms of the production function contain the same information. But for our purposes the capital-output ratio form is more convenient to work with, so work with it we shall.

What, then, is the equilibrium value of output per worker? We know the value of the diminishing-returns-to-investment parameter  $\alpha$ . We know the value of the efficiency of labor  $E$ . And we know that in equilibrium  $K/Y$  will be equal to  $s/\delta$ . So just substitute  $s/\delta$  in for  $K/Y$  where it appears, and calculate. Box 4.3 shows how.



#### CALCULATING OUTPUT PER WORKER: AN EXAMPLE

Suppose we know that the current value of the efficiency of labor  $E$  is \$10,000 a year, and that the diminishing-returns-to-investment parameter  $\alpha$  is 0.5. Then — in this special case in which both the labor force and the efficiency of labor are constant — we can calculate what the long-run value of output per worker will be once we know the saving-investment rate  $s$  and the depreciation rate  $\delta$ .

Suppose the economy's saving-investment rate  $s$  is 16 percent, and the depreciation rate  $\delta$  is 0.04. Then substituting the parameters into the capital-output ratio form of the production function is straightforward:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} (E)$$

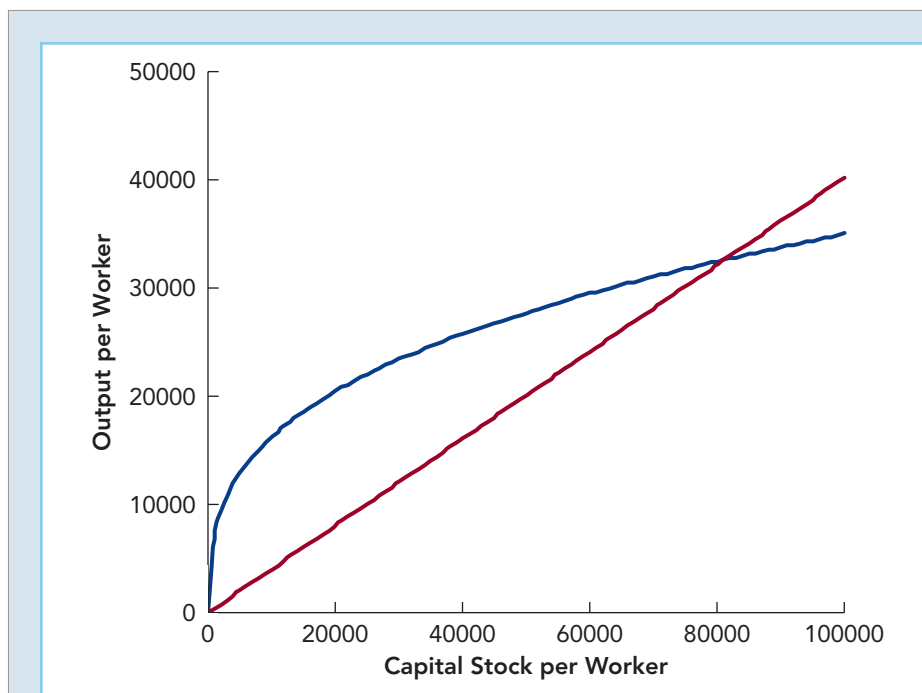
$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{0.5}{1-0.5}} (10,000)$$

And we know that in the very long run the capital-output ratio  $K/Y$  will be  $s/\delta = .16/.04 = 4$ . Thus

$$\frac{Y}{L} = (4)^{\frac{0.5}{1-0.5}} (10,000)$$

Output per worker will be \$40,000 per year. 

We can reach the same conclusion by a different, graphical, approach — an approach that is more friendly to those who prefer lines and curves on graphs to Greek variables and exponents in equations. Recall Figure 4.2, which showed how to draw the production function at any one moment in time (for a fixed and known level of the efficiency of labor  $E$ ). Plot output per worker  $Y/L$  on the vertical axis,

**FIGURE 4.5****Equilibrium with Just Investment and Depreciation**

The blue line shows output per worker as a function of the capital stock per worker for the known, constant values of the diminishing returns to investment parameter  $\alpha$  and the efficiency of labor  $E$ . The pink line shows the reciprocal of the equilibrium capital-output ratio  $\delta/s$ . If the economy is to the left of where the lines cross, the capital stock is growing. If the economy is to the right, the capital stock is shrinking. Where the lines cross is the equilibrium value of output per worker and capital per worker.

and the capital stock per worker  $K/L$  on the horizontal axis. This curve tells you the relationship between levels of capital per worker and what the economy can produce.

Which of the many points on this production function curve is the economy's equilibrium? Recall our equilibrium condition: In equilibrium the economy's capital intensity, its capital-output ratio  $K/Y$ , is equal to  $s/\delta$ . We can think of this equilibrium condition  $K/Y = s/\delta$  as another line on the figure. It is a straight line starting at the bottom left (0,0) origin point and climbing to the upper right, with its slope  $Y/K$  equal to  $\delta/s$ . Figure 4.5 shows that only one point satisfies both (1) this equilibrium condition for the economy's capital intensity and (2) the behavioral relationship relating output per worker to capital per worker, that is, the production function. This point is where the curves in Figure 4.5 cross.

We can use either the algebra or the graphical method to think about the long-run consequences of, say, changes in a government's fiscal policy. Before the 2000 presidential election the U.S. federal budget was in surplus and was expected to remain in surplus for decades. In the following few years, however, the George W. Bush administration pushed hard for significant increases in spending (which drove conservative small-government Republicans up the wall) and significant decreases in taxes (which drove liberal pro-government-program Democrats around the bend), with the result being a large downward shift in government saving  $S^G$  and thus in the economy's saving-investment rate  $s$ . Such a shift is going to mean that the economy's equilibrium condition  $K/Y = s/\delta$  will be satisfied at a lower capital intensity, and so the economy will be poorer. The equilibrium line rotates counterclockwise, and so the intersection point moves to a lower equilibrium value of output per worker.

This depressing effect of rising government deficits on the standard of living is one important reason that international agencies like the International Monetary Fund (IMF) and the World Bank and almost all economists advise governments to avoid large and prolonged deficits.

Note that in this particular, restricted case the economy's labor force is constant. Its capital stock is constant. There are no changes in the efficiency of labor. Thus equilibrium output per worker is constant. There is — in this particular, restricted case — no growth of output per worker when the economy is in equilibrium. If we are to have a model in which economic growth continues, then we need to have growth in the labor force and, more important, growth in the efficiency of labor — which is why we have to move on to the next subsection, and think about a more complicated model.

### Adding in Labor-Force and Labor-Efficiency Growth

If labor forces were constant and technological and organizational progress nonexistent, we could stop the chapter here. But the economy's labor force continues to grow as more people turn 18 and join the labor force than retire and immigrants continue to arrive. And the efficiency of labor rises as science and technology progress and people keep thinking of new and more efficient forms of business organization.

We assume — once again making a simplifying leap — that the economy's *labor force*  $L$  is growing at a constant proportional rate  $n$  every year. (Note that  $n$  is not the same across countries and can shift over time in any one country, but our background assumption will be that  $n$  is constant as far as we can see into the future.) Thus between this year and the next the labor force grows according to the formula

$$L_{t+1} = (1 + n)L_t$$

Next year's labor force will be  $n$  percent higher than this year's labor force, as Figure 4.6 shows. If this year's labor force is 10 million and its growth rate  $n$  is 2 percent per year, then next year's labor force will be

$$L_{t+1} = (1 + n)L_t$$

$$L_{t+1} = (1 + 2\%)(10,000,000)$$

$$L_{t+1} = (1.02)(10,000,000)$$

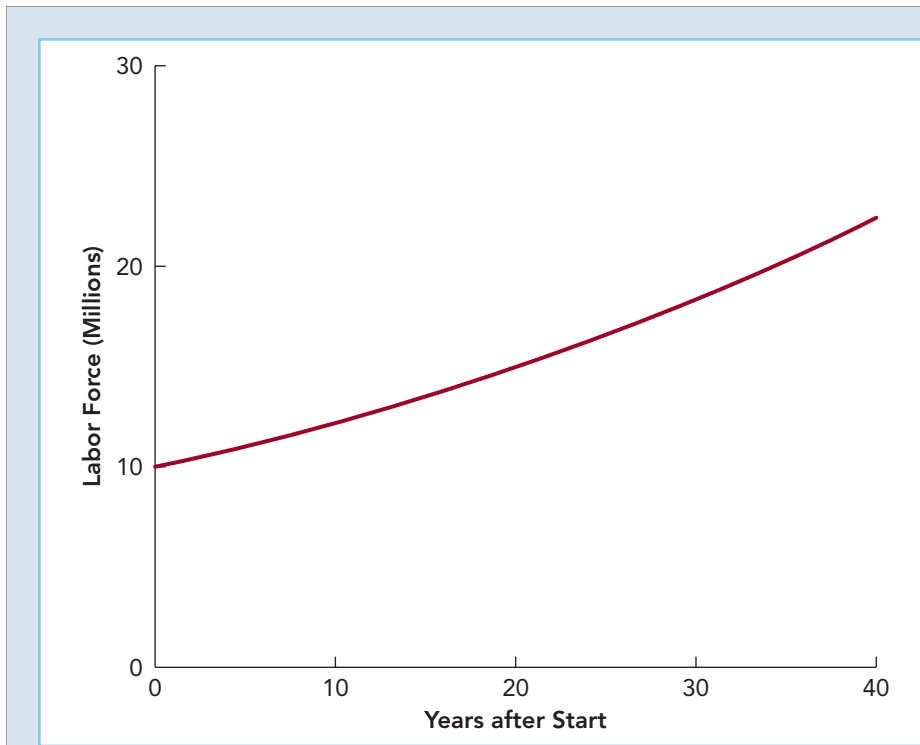
$$L_{t+1} = 10,200,000$$

We assume that the rate of growth of the labor force is constant not because we believe that labor-force growth is unchanging, but because the assumption makes the analysis of the model simpler. The trade-off between realism in the model's description of the world and simplicity as a way to make the model easier to analyze is one that economists always face, and economists have a strong bias toward resolving this trade-off in favor of simplicity.

We also assume that the efficiency of labor  $E$  grows at a constant proportional rate  $g$ :

$$E_{t+1} = (1 + g)E_t$$

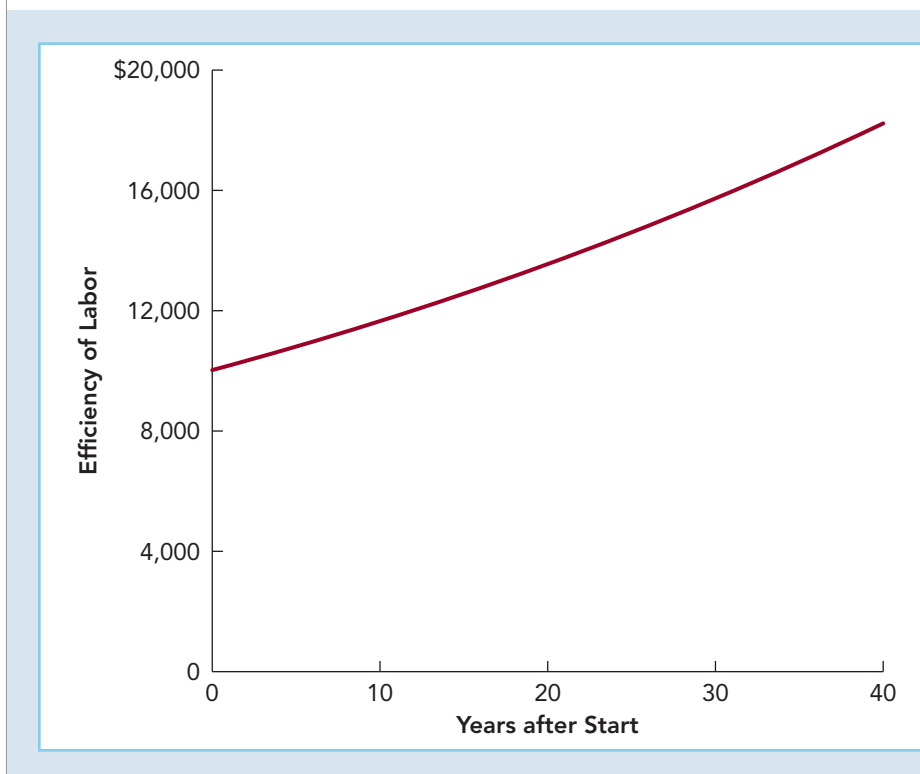
Next year's level of the efficiency of labor will be  $g$  percent higher than this year's level, as Figure 4.7 shows. If this year the efficiency of labor is \$10,000 per worker,



**FIGURE 4.6**

**Constant Labor-Force Growth at  $n = 2$  Percent per Year**

The labor force growing at 2 percent per year. A labor force increasing at 2 percent per year will double roughly every 35 years.



**FIGURE 4.7**

**Efficiency-of-Labor Growth at  $g = 1.5$  Percent per Year**

Labor efficiency growing at 1.5 percent per year. If the efficiency of labor grows at 1.5 percent per year, it will take roughly 47 years for it to double.

and if  $g$  is 1.5 percent per year, then next year the efficiency of labor will be

$$E_{t+1} = (1 + g)E_t$$

$$E_{t+1} = (1 + 0.015)(\$10,000)$$

$$E_{t+1} = \$10,150$$

### The Balanced-Growth Capital-Output Ratio

Earlier, when we had assumed that labor force and efficiency were both constant, and thus that  $n$  and  $g$  were both equal to 0, our equilibrium condition was  $K/Y = s/\delta$ . Since the saving-investment rate  $s$  and the depreciation rate  $\delta$  are both constant, our equilibrium condition required that the capital-output ratio  $K/Y$  be constant. Now we've added realism to our model by allowing the labor force and efficiency to both increase over time at constant rates  $n$  and  $g$ . What effect does this added realism — allowing  $n$  and  $g$  to take on values other than 0 — have on our equilibrium condition?

In an important sense, none! Once again, our equilibrium condition is that the capital-output ratio be constant. When the capital-output ratio is constant, we say that the economy is in **balanced growth**: Output per worker is then growing at the same rate as the capital stock per worker. The two variables are in balance, and they are also both growing at the same rate as the efficiency of labor.

But at what value will the economy's capital-output ratio be constant? Here is where allowing  $n$  and  $g$  to take on values other than 0 matters. The capital-output ratio will be constant — and therefore we'll be in balanced-growth equilibrium — when  $K/Y = s/(n + g + \delta)$ . Add up the economy's labor-force growth rate, efficiency of labor growth rate, and depreciation rate; divide the saving-investment rate by that sum; and that is your **balanced-growth equilibrium capital-output ratio**.

Why is  $s/(n + g + \delta)$  the capital-output ratio in equilibrium? Think of it this way: Suppose the economy is in balanced growth. How much is it investing? There must be investment equal to  $\delta K$  to replace depreciated capital. There must be investment equal to  $nK$  to provide the labor force which is expanding at rate  $n$  with the capital it will need. And since the efficiency of labor is growing at rate  $g$ , there must be investment equal to  $gK$  in order for the capital stock to keep up with increasing efficiency of labor. Adding these three parts of required investment together and setting the sum equal to the investment  $sY$  actually going on,

$$(n + g + \delta)K = sY$$

shows clearly that the economy's investment requirements for balanced growth equal the actual flow of investment when

$$\frac{K}{Y} = \frac{s}{(n + g + \delta)}$$

This is the balanced-growth equilibrium condition. It is constant because  $s$ ,  $n$ ,  $g$ , and  $\delta$  are all constant. So when there is balanced growth — when output per worker  $Y/L$  and capital per worker  $K/L$  are growing at the same rate — the capital-output ratio  $K/Y$  will be constant.

To see more formally that  $K/Y = s/(n + g + \delta)$  is the balanced-growth equilibrium condition requires a short march through simple algebra. Take a look again

#### balanced growth

When a country has a capital-output ratio equal to its equilibrium value,  $s/(n + g + \delta)$ .

#### balanced-growth equilibrium capital-output ratio

The value of the capital-output ratio to which an economy with constant saving rate, depreciation rate, labor-force growth rate, and efficiency growth rate converges over time. Equal to  $s/(n + g + \delta)$ .

at the capital-output ratio form of the production function:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{1-\alpha} (E)$$

Recall that the efficiency of labor  $E$  is growing at the constant proportional rate  $g$ . With the capital-output ratio  $K/Y$  constant, output per worker must be growing at the rate  $g$  as well. Recall also that the labor force is growing at a constant proportional rate  $n$ . With output per worker growing at rate  $g$  and the number of workers growing at rate  $n$ , total output is growing at the constant rate  $n + g$ .<sup>5</sup> For the capital-output ratio  $K/Y$  to be constant, the capital stock also has to be growing at rate  $n + g$ . This means that the year-over-year change  $\Delta K_{t+1}$  in the capital stock must be

$$\Delta K_{t+1} = (n + g)K_t$$

#### SOME MATHEMATICAL RULES OF THUMB: TOOLS

This is a good place to introduce four mathematical rules of thumb to make life easier throughout this book. They are all only *approximations*. But they are good enough for our purposes. They are:

1. *The growth-of-a-product rule:* The growth rate of a *product* is equal to the *sum* of the growth rates of its components. Since total output  $Y$  is equal to output per worker  $Y/L$  times the number of workers  $L$ , the growth rate of total output  $Y$  will be equal to the growth rate of  $Y/L$  plus the growth rate of  $L$ .
2. *The growth-of-a-quotient rule:* The proportional change of a *quotient* is equal to the *difference* between the proportional changes of its components. Since output per worker  $Y/L$  is equal to the quotient of output  $Y$  and the number of workers  $L$ , its growth rate will be the difference between their growth rates.
3. *The growth-of-a-power rule:* The proportional change of a *quantity raised to a power* is equal to the *proportional change in the quantity times the power* to which it is raised. For example, suppose that we have a situation in which output  $Y$  is equal to the capital stock  $K$  raised to the power  $\alpha$ :  $Y = K^\alpha$ . Then the growth rate of  $Y$  will be equal to  $\alpha$  times the growth rate of  $K$ .
4. *The rule of 72:* A quantity growing at  $k$  percent per year doubles in  $72/k$  years. A quantity shrinking at  $k$  percent per year halves itself in  $72/k$  years.

You may hear people say that a background in calculus is needed to understand intermediate macroeconomics. That is not true. These four mathematical rules of thumb contain 95 percent of what calculus is used for in intermediate macroeconomics. (Of course, you do need calculus if you want to do more than just take them on faith, and instead have a deep understanding of just *why* they work.)

<sup>5</sup>The fact that the growth rate of a product,  $Y/L$  times  $L$ , is equal to the sum of the growth rates, growth rate of  $Y/L$  plus the growth rate of  $L$ , is one of the handy mathematical rules of thumb contained in Box 4.4.



Since next year's capital stock equals this year's plus investment minus depreciation, the year-over-year change in the capital stock is also equal to

$$\Delta K_t = sY_t - \delta K_t$$

Setting our two expressions for the change in the capital stock equal to each other,

$$(n + g)K_t = sY_t - \delta K_t$$

collecting the terms with the capital stock on the left-hand side,

$$(n + g + \delta)K_t = sY_t$$

and then dividing,

$$\frac{K_t}{Y_t} = \frac{s}{n + g + \delta}$$

gives us the Solow growth model's equilibrium condition:  $K/Y = s/(n + g + \delta)$ . The balanced-growth equilibrium capital-output ratio is equal to the share of production that is saved and invested for the future — the economy's saving-investment rate  $s$  — divided by the sum of three things:

- The proportional rate of growth of the labor force  $n$ .
- The proportional growth rate of the efficiency of labor  $g$ .
- The depreciation rate  $\delta$  at which capital breaks down and wears out.

We'll sometimes call  $s/(n + g + \delta)$  the “equilibrium” capital-output ratio, and we'll sometimes call it the “balanced-growth” capital-output ratio. To be always saying “balanced-growth equilibrium” is too much of a mouthful.

How do we know  $K/Y = s/(n + g + \delta)$  gives us balanced growth, where capital per worker  $K/L$  and output per worker  $Y/L$  grow at the same rate? Suppose the current capital-output ratio is lower than  $s/(n + g + \delta)$ . Then  $sY$  will be more than  $(n + g + \delta)K$  — the economy's total investment which is equal to  $sY$ , the saving rate  $s$  times the level of output  $Y$ . Thus saving will more than provide new workers with the capital they need to be fully productive, more than cover the increase in output due to the increase in labor efficiency, and more than compensate for the wearing out of capital through depreciation. The capital stock will grow faster than  $n + g$ . Since  $n + g$  is the rate at which output grows, the capital-output ratio will rise.

Suppose instead the current capital-output ratio is above  $s/(n + g + \delta)$ . Then  $sY$  will be less than  $(n + g + \delta)K$  — the economy's total investment  $sY$  will be insufficient to keep the capital stock growing at rate  $n + g$ . And since  $n + g$  is the rate at which output grows, the capital-output ratio will fall.

Thus a capital-output ratio greater than  $s/(n + g + \delta)$  makes the capital-output ratio fall. And a capital-output ratio less than  $s/(n + g + \delta)$  makes the capital-output ratio rise. So a capital-output ratio equal to  $s/(n + g + \delta)$  is indeed the balanced-growth equilibrium condition.

We now have our Solow growth model. It consists of one equilibrium condition telling us that the stable capital-output ratio will be  $K/Y = s/(n + g + \delta)$ . It consists of a production function. And it consists of four assumptions:

- The rate of labor-force growth equals  $n$ .
- The rate of increase in the efficiency of labor equals  $g$ .
- The rate of depreciation equals  $\delta$ .
- The saving-investment rate equals  $s$ .

“Robert Solow got the Nobel Prize for that?!” you may ask. Ah, but what he got the Nobel Prize for was taking a complicated subject and making a useful model of it that was very simple indeed. The model is simple to write down. But it is powerful. As we unfold its implications and use it to understand very long run economic growth, we will see that it generates many insights.

#### RECAP THE SOLOW GROWTH MODEL

When the economy’s capital stock and its level of real GDP are growing at the same proportional rate, its capital-output ratio — the ratio of the economy’s capital stock  $K$  to annual real GDP  $Y$  — is constant, and the economy is in balanced-growth equilibrium. In equilibrium, the capital-output ratio  $K/Y$  will equal the constant ratio  $s/(n + g + \delta)$ . The standard growth model analyzes how this balanced-growth equilibrium is determined by four factors: the economy’s saving-investment rate  $s$ , the economy’s labor-force growth rate  $n$ , the growth rate of the efficiency of labor  $g$ , and the capital stock depreciation rate  $\delta$ .

### 4.3 UNDERSTANDING THE GROWTH MODEL

#### Balanced-Growth Output per Worker

Suppose that the capital-output ratio is equal to its balanced-growth equilibrium value: The economy is on its *balanced-growth path*. What does an economy on its balanced-growth path look like? The first and most important thing to look at is output per worker  $Y/L$  — what it is now, and how it grows.  $Y/L$  is, after all, our best simple proxy for the economy’s overall level of prosperity: for material standards of living and for the possession by the economy of the resources needed to diminish poverty. Let’s calculate the level of output per worker  $Y/L$  along the balanced-growth path (paying close attention to the time subscripts, for we are interested in where the economy is, where it was, and where it will be).

Begin with the capital-output ratio version of the production function:

$$\frac{Y_t}{L_t} = \left( \frac{K_t}{Y_t} \right)^{1-\alpha} (E_t)$$

Since the economy is on its balanced-growth path, it satisfies the equilibrium condition  $K/Y = s/(n + g + \delta)$ :

$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{1-\alpha} (E_t)$$

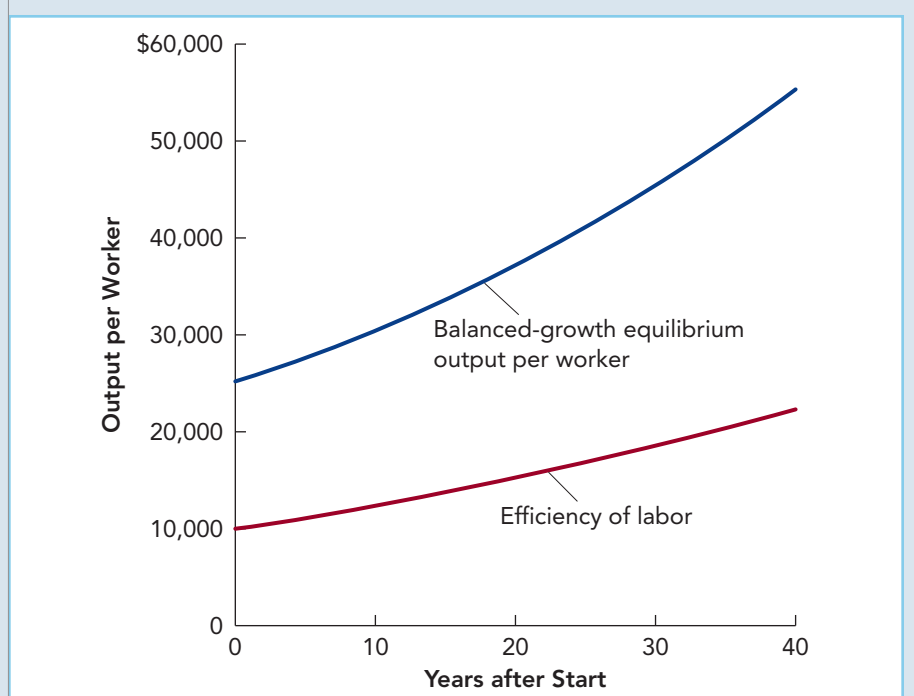
But  $s$ ,  $n$ ,  $g$ ,  $\delta$ , and  $\alpha$  are all constants, and so  $[s/(n + g + \delta)]^{1-\alpha}$  is a constant. This tells us that along the balanced-growth path, output per worker is simply a constant multiple of the efficiency of labor, with the multiple equal to

$$\left( \frac{s}{n + g + \delta} \right)^{1-\alpha}$$

Over time, the efficiency of labor grows. Each year it is  $g$  percent higher than the last year. Since along the balanced-growth path output per worker  $Y/L$  is just a

**FIGURE 4.8****Balanced Growth:  
Output per Worker and  
the Efficiency of Labor**

Along its balanced-growth path, the level of output per worker is a constant multiple of the efficiency of labor. What that multiple is depends on all the parameters of the growth model: the saving rate  $s$ , the labor-force growth rate  $n$ , the efficiency-of-labor growth rate  $g$ , the depreciation rate  $\delta$ , and the diminishing-returns-to-investment parameter  $\alpha$ .



constant multiple of the efficiency of labor, it too must be growing at the same proportional rate  $g$ . Figure 4.8 shows the pattern of balanced-growth output per worker and the efficiency of labor.

We now see how capital intensity and technological and organizational progress drive economic growth. Capital intensity — the economy's capital-output ratio — determines what multiple output per worker  $Y/L$  is of the current efficiency of labor  $E$ . Things that increase capital intensity — raise the capital-output ratio — make balanced-growth output per worker a higher multiple of the efficiency of labor, and so make the economy richer. Things that reduce capital intensity make balanced-growth output per worker a lower multiple of the efficiency of labor, and so make the economy poorer.

Suppose that  $\alpha$  is  $1/2$ , so that  $\alpha/(1 - \alpha)$  is 1, and that  $s = n + g + \delta$ , so that the balanced-growth capital-output ratio is 1. Then balanced-growth output per worker is simply equal to the efficiency of labor. But if we consider another economy with twice the saving rate  $s$ , its balanced-growth capital-output ratio is 2, and its balanced-growth level of output per worker is twice the level of the efficiency of labor.

The higher is the parameter  $\alpha$  — that is, the slower diminishing returns to investment set in — the stronger is the effect of changes in the economy's balanced-growth capital intensity on the level of output per worker.

- Suppose that the balanced-growth capital-output ratio is 4. Then if  $\alpha$  is  $1/3$ ,  $\alpha/(1 - \alpha)$  is  $1/2$ , and the level of output per worker is twice the level of the efficiency of labor. Economists think that  $1/3$  is a reasonable parameter value for the United States today.

- By contrast, if  $\alpha$  is  $1/2$ ,  $\alpha/(1 - \alpha)$  is equal to 1, and again with a balanced-growth capital-output ratio of 4, the level of output per worker is fully four times the level of the efficiency of labor. Economists think that  $1/2$  is a reasonable parameter value for the United States a century ago or for relatively poor countries today.

Note — this is important — that changes in the economy's capital intensity shift the balanced-growth path up or down to a different multiple of the efficiency of labor, but the growth rate of  $Y/L$  along the balanced-growth path is simply the rate of growth  $g$  of the efficiency of labor  $E$ . The material standard of living grows at the same rate as labor efficiency. To change the very long-run growth rate of the economy you need to change how fast the efficiency of labor grows. Changes in the economy that merely alter the capital-output ratio will not do it.

This is what tells us that technology, organization, worker skills — all those things that increase the efficiency of labor and keep on increasing it — are ultimately more important to growth in output per worker than saving and investment. The U.S. economy experienced a large increase in its capital-output ratio in the late nineteenth century. It may be experiencing a similar increase now, as we invest more and more in computers. But the Gilded Age industrialization came to an end, and the information technology revolution will run its course. Aside from these episodes, it is growth in the efficiency of labor  $E$  that sustains and accounts for the lion's share of long-run economic growth.

We are now finished with our analysis of the economy's balanced-growth path. We see that calculating output per worker when the economy is on its balanced-growth path is a straightforward three-step procedure:

1. Calculate the balanced-growth equilibrium capital-output ratio  $s/(n + g + \delta)$ , the saving rate divided by the sum of the labor-force growth rate, the efficiency of labor growth rate, and the depreciation rate.
2. Raise the balanced-growth capital-output ratio to the  $\alpha/(1 - \alpha)$  power, where  $\alpha$  is the diminishing-returns-to-investment parameter in the production function.
3. Multiply the result by the current value of the efficiency of labor  $E$ .

The result is the value of output per worker:

$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}} (E_t)$$

An example is given in Box 4.5.

#### CALCULATING EQUILIBRIUM OUTPUT PER WORKER: AN EXAMPLE

To see how to use the expression for output per worker when the economy is on its balanced-growth path,

$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}} (E_t)$$

let's work through an example. Suppose that the economy's labor-force growth rate  $n$  is 1 percent per year, the efficiency-of-labor growth rate  $g$  is 2 percent per year, and the depreciation rate  $\delta$  is 3 percent per year. Suppose further that the diminishing-returns-to-investment parameter  $\alpha$  is  $1/2$ , and the economy's saving-investment rate  $s$  is 18 percent.



Then the balanced-growth equilibrium capital-output ratio  $s/(n + g + \delta)$  equals 3, and  $\alpha/(1 - \alpha)$  equals 1. Substituting these values into the equation above,

$$\frac{Y_t}{L_t} = (3)^1 (E_t)$$

$$\frac{Y_t}{L_t} = 3E_t$$

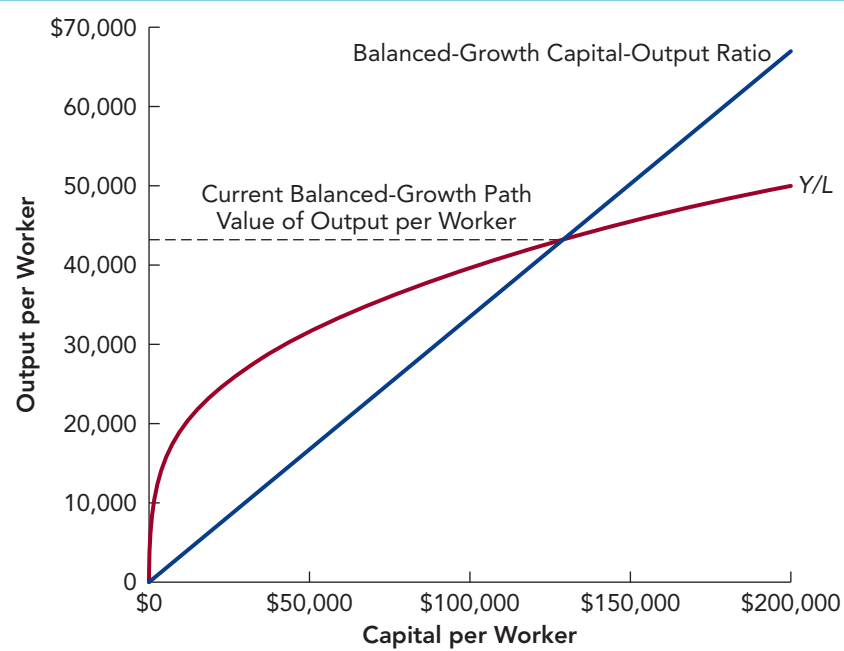
For these parameter values, balanced-growth output per worker is simply three times the efficiency of labor, whatever the value of the efficiency of labor is. When the efficiency of labor is \$10,000 per year, balanced-growth output per worker is \$30,000 per year. When the efficiency of labor is \$20,000 per year, balanced-growth output per worker is \$60,000 per year.

Over time, because balanced-growth output per worker is a constant multiple of the efficiency of labor, its growth rate is the same as the growth rate of the efficiency of labor  $g$ , 2 percent per year. ◆

The implications of the balanced-growth capital-output ratio for the balanced-growth level of output per worker, and how that level changes over time, can be seen in an alternative, diagrammatic way. Take a look at Figure 4.9. As in Figure 4.5, draw the production-function curve that shows output per worker  $Y/L$  as a function of capital per worker  $K/L$  for the current level of the efficiency of labor  $E_t$ . In addition, as in Figure 4.5, draw the line that shows where the capital-output ratio is equal to its balanced-growth equilibrium value,  $K/Y = s/(n + g + \delta)$ . This line

**FIGURE 4.9**  
Calculating Balanced-Growth Output per Worker

The economy is on its balanced-growth path at that level of capital per worker and output per worker at which the capital-output ratio is equal to its equilibrium value.



starts at the bottom left origin point  $(0, 0)$  and climbs toward the upper right. Because  $K/L$  is on the horizontal axis and  $Y/L$  is on the vertical axis, the slope of the line is  $Y/K$  or  $(n + g + \delta)/s$ .

Look once again at where the curves cross. That point shows the current level of output per worker is along the balanced-growth path. Output per worker is given by the production function for the current levels of capital per worker and the efficiency of labor. And the capital-output ratio is at its balanced-growth path level. Anything that increases the balanced-growth capital-output ratio will rotate the capital-output ratio line clockwise. Thus it will raise the balanced-growth path level of output per worker. Anything that decreases the balanced-growth capital output ratio rotates the capital-output ratio line counterclockwise. It thus lowers the level of output per worker for the given value of the efficiency of labor  $E$ .

Over time the efficiency of labor increases. As the efficiency of labor increases, the production-function curve in Figure 4.9 will shift up and out to the right. Over time, therefore, the balanced-growth path equilibrium levels of output per worker and capital per worker levels will rise as the economy climbs up and to the right along the constant capital-intensity line.

### Off the Balanced-Growth Path

What if the economy is not on its balanced-growth path? How can we use a model which assumes that the economy is on its balanced-growth path to analyze a situation in which the economy is *not* on that path? We still can use the model — and this is an important part of the magic of economics — because being on the balanced-growth path is an *equilibrium condition*. In an economic model, the thing to do if an equilibrium condition is not satisfied is to wait and, after a while, look again. When we look again, it will be satisfied.

Whenever the capital-output ratio  $K/Y$  is above its balanced-growth equilibrium value  $s/(n + g + \delta)$ ,  $K/Y$  is falling: Investment is insufficient to keep the capital stock growing as fast as output. Whenever  $K/Y$  is below its balanced-growth equilibrium value,  $K/Y$  is rising: Capital stock growth outruns output. Figure 4.10 gives an indication of how this process of **convergence** to the balanced-growth value proceeds. And as the capital-output ratio converges to its balanced-growth value, so does the economy's level of output per worker converge to its balanced-growth path.

The fact that an economy converges to its balanced-growth path makes analyzing the long-run growth of an economy not on its balanced-growth path relatively easy as well:

1. Calculate the balanced-growth path.
2. From the balanced-growth path, forecast the future of the economy: If the economy is on its balanced growth path today, it will stay on that path in the future (unless some of the parameters —  $n$ ,  $g$ ,  $\delta$ ,  $s$ , and  $\alpha$  — change). If the economy is not on its balanced growth path today, it is heading for that path and will get there eventually. But as Figure 4.10 points out, decades may pass before an economy finally gets on that balanced-growth path.

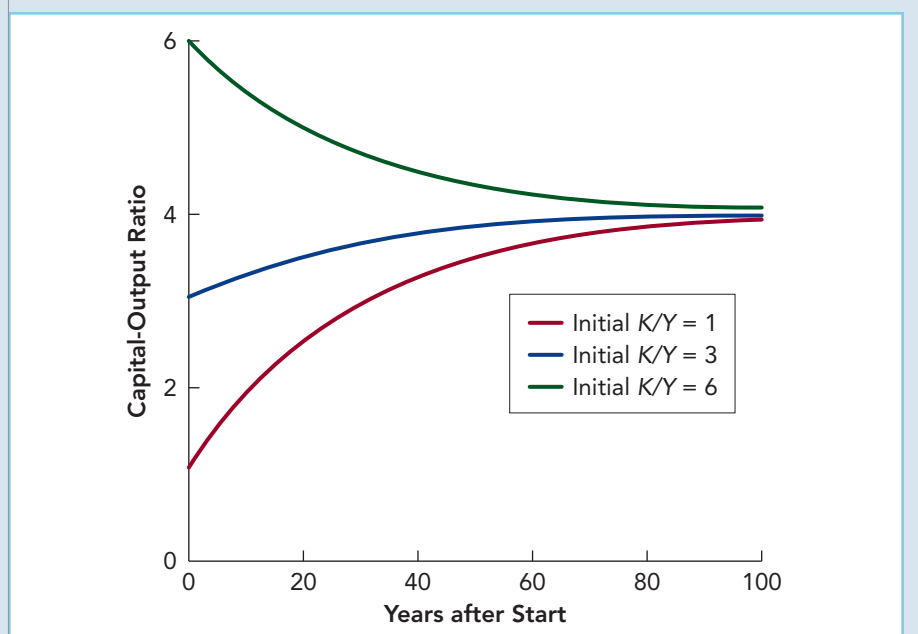
Thus economic forecasting becomes simple. All you have to do is predict that the economy will head for its balanced-growth path, and calculate what the balanced growth path is.

#### convergence

The tendency for a country to approach its balanced-growth path with a constant capital-output ratio.

**FIGURE 4.10****Convergence to a Balanced-Growth Capital-Output Ratio of 4**

Suppose the balanced-growth equilibrium capital-output ratio is 4. Whether the capital-output ratio starts above or below its balanced-growth equilibrium value, it converges to the level equal to  $s/(n + g + \delta)$ .

**How Fast the Economy Heads for Its Balanced-Growth Path**

How fast does an economy head for its balanced-growth path? We assert — but will not derive — that a fraction  $(1 - \alpha)(n + g + \delta)$  of the gap between its balanced-growth path will be closed each year. If  $(1 - \alpha)(n + g + \delta)$  turns out to be equal to 0.04, the capital-output ratio will close approximately 4 percent of the gap between its current level and its balanced-growth value in a year. According to the *rule of 72* (see Box 4.4) an economy closing 4 percent of the gap between its current and its equilibrium value each year will move halfway to equilibrium in  $72/4$ , or 18, years. An example is given in Box 4.6. The convergence of an economy following the Solow growth model to its balanced-growth path does not happen overnight or in a year. It is a matter of decades. The Solow growth model is definitely a long-run model.

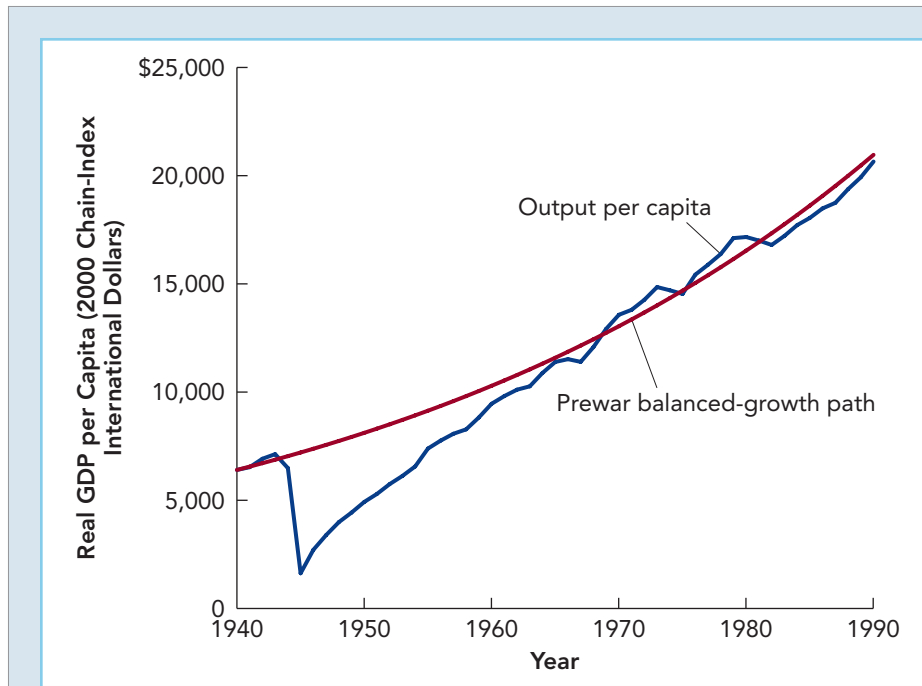
**CONVERGING TO THE BALANCED-GROWTH PATH: AN EXAMPLE**

Consider an economy in which the rate of labor-force growth  $n = 1$  percent per year, in which the efficiency of labor grows at a rate  $g = 2$  percent per year, in which the depreciation rate  $\delta = 5$  percent per year, and in which the diminishing-returns-to-investment parameter  $\alpha = 1/2$ .

This economy will, according to the Solow growth model, each year close a fraction of the gap between its current capital-output ratio and its balanced-growth capital-output ratio equal to

$$(1 - \alpha)(n + g + \delta) = (1 - 1/2)(0.01 + 0.02 + 0.05) = 0.04$$

or 4 percent per year. According to the rule of 72, the economy will close half of the gap to its balanced-growth path in 18 years. ◆

**FIGURE 4.11****The Return of the West German Economy to Its Balanced-Growth Path**

Economies do converge to and then remain on their balanced-growth paths. The West German economy after World War II is a case in point.

Source: J. Bradford DeLong and Barry Eichengreen, "The Marshall Plan: History's Most Successful Structural Adjustment Programme," in Rüdiger Dornbusch, Wilhelm Nolling, and Richard Layard, eds., *Postwar Economic Reconstruction and Lessons for the East Today* (Cambridge: MIT Press, 1993), pp. 189–230.

We can see such convergence in action in many places and times. For example, consider the post–World War II history of West Germany. The defeat of the Nazis left the German economy at the end of World War II in ruins. Output per worker was less than one-third of its prewar level. The economy's capital stock had been wrecked and devastated by three years of American and British bombing and then by the ground campaigns of the last six months of the war. But in the years immediately after the war, the West German economy's capital-output ratio rapidly grew and converged back to its prewar value. As Figure 4.11 shows, within 12 years the West German economy had closed half the gap back to its pre–World War II growth path. And within 30 years the West German economy had effectively closed the entire gap between where it had started at the end of World War II and its balanced-growth path.

**RECAP UNDERSTANDING THE GROWTH MODEL**

According to the Solow growth model, capital intensity and growth in the efficiency of labor together determine the destiny of an economy. The value of the equilibrium balanced-growth capital-output ratio and the economy's diminishing-returns-to-investment parameter determine the multiple that balanced-growth output per worker is of the current efficiency of labor. The growth rate of output per worker along the economy's balanced growth path is equal to the growth rate of the efficiency of labor. And if the economy is not on its balanced-growth path, the Solow growth model tells us that it is converging to it — although this convergence takes decades, not years.

#### 4.4 USING THE SOLOW GROWTH MODEL

Up until now we have assumed that all the parameters of the Solow growth model are unchanging. But what if one or more of them were to shift? What if the labor-force growth rate were to rise, or the rate of technological progress to fall? The principal use of the Solow growth model is to analyze questions like these: how changes in the economic environment and in economic policy will affect an economy's long-run levels and growth path of output per worker  $Y/L$ .

Let's consider, as examples, several such shifts: an increase in the growth rate of the labor force  $n$ , a change in the economy's saving-investment rate  $s$ , and a change in the growth rate of labor efficiency  $g$ . All of these will have effects on the balanced-growth path level of output per worker. But only one — the change in the growth rate of labor efficiency — will permanently affect the growth rate of the economy.

##### The Labor-Force Growth Rate

Real-world economies exhibit profound shifts in labor-force growth. The average woman in India today has only half the number of children that the average woman in India had only half a century ago. The U.S. labor force in the early eighteenth century grew at nearly 3 percent per year, doubling every 24 years. Today the U.S. labor force grows at 1 percent per year. Changes in the level of prosperity, changes in the freedom of migration, changes in the status of women that open up new categories of jobs to them (Supreme Court Justice Sandra Day O'Connor could not get a private-sector legal job in San Francisco when she graduated from Stanford Law School even with her amazingly high class rank), changes in the status of women or the average age of marriage or the availability of birth control that change fertility — all of these have powerful effects on economies' rates of labor-force growth.

What effects do such changes have on output per worker  $Y/L$  — on our measure of material prosperity? The faster the growth rate of the labor force  $n$ , the lower will be the economy's balanced-growth capital-output ratio  $s/(n + g + \delta)$ . Why? Because each new worker who joins the labor force must be equipped with enough capital to be productive and to, on average, match the productivity of his or her peers. The faster the rate of growth of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker on the balanced-growth path. How large will the long-run change in the level of output be, relative to what would have happened had labor-force growth not increased? It is straightforward to calculate if we know the other parameter values, as is shown in Box 4.7.

How important is all this in the real world? Does a high rate of labor-force growth play a role in making countries relatively poor not just in economists' models but in reality? It turns out that it is important, as Figure 4.12 shows. Of the 22 countries in the world with output-per-worker levels at least half of the U.S. level, 18 have labor-force growth rates of less than 2 percent per year, and 12 have labor-force growth rates of less than 1 percent per year. The additional investment requirements imposed by rapid labor-force growth are a powerful reducer of capital intensity and a powerful obstacle to rapid economic growth.

**AN INCREASE IN THE LABOR-FORCE GROWTH RATE: AN EXAMPLE**

Consider an economy in which the parameter  $\alpha$  is  $1/2$ , the efficiency of labor growth rate  $g$  is 1.5 percent per year, the depreciation rate  $\delta$  is 3.5 percent per year, and the saving rate  $s$  is 21 percent. Suppose that the labor-force growth rate suddenly and permanently increases from 1 to 2 percent per year.

Before the increase in the labor-force growth rate, the balanced-growth equilibrium capital-output ratio was

$$\frac{s}{n + g + \delta} = \frac{0.21}{0.01 + 0.015 + 0.035} = \frac{0.21}{0.06} = 3.5$$

After the increase in the labor-force growth rate, the new balanced-growth equilibrium capital-output ratio will be

$$\frac{s}{n + g + \delta} = \frac{0.21}{0.02 + 0.015 + 0.035} = \frac{0.21}{0.07} = 3$$


Before the increase in labor-force growth, the level of output per worker along the balanced-growth path was equal to

$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\left( \frac{\alpha}{1-\alpha} \right)} (E_t) = (3.5)^{\left( \frac{1/2}{1-1/2} \right)} = (3.5)^1 (E_t)$$

After the increase in labor-force growth, the level of output per worker along the balanced-growth path will be equal to

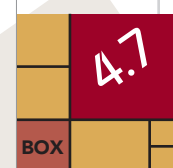
$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\left( \frac{\alpha}{1-\alpha} \right)} (E_t) = (3)^{\left( \frac{1/2}{1-1/2} \right)} = (3)^1 (E_t)$$

This fall in the balanced-growth path level of output per worker means that in the very long run — after the economy has converged to its new balanced-growth path — one-seventh of its economic prosperity has been lost because of the increase in the rate of labor-force growth.

In the short run of a year or two, however, such an increase in the labor-force growth rate has little effect on output per worker. In the months and years after labor-force growth increases, the increased rate of labor-force growth has had no time to affect the economy's capital-output ratio. But over decades and generations, the capital-output ratio will fall as it converges to its new balanced-growth equilibrium level. 

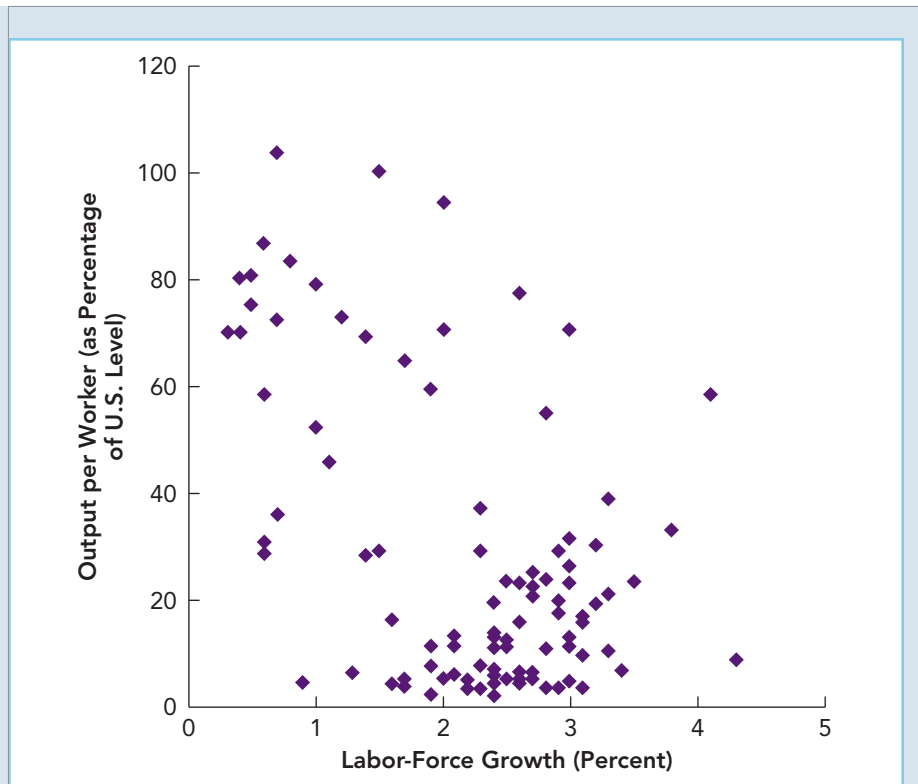
A sudden and permanent change in the rate of growth of the labor force will immediately and substantially change the level of output per worker along the economy's balanced-growth path: It will shift the balanced-growth path for output per worker up (if labor-force growth falls) or down (if labor-force growth rises). But there is no corresponding immediate jump in the actual level of output per worker in the economy. Output per worker doesn't immediately jump — it is just that the shift in the balanced-growth path means that the economy is no longer in its Solow growth model long-run equilibrium.

It takes time, decades and generations, for the economy to converge to its new balanced-growth path equilibrium, and thus for the shift in labor force



**FIGURE 4.12****The Labor-Force Growth Rate Matters**

The average country with a labor-force growth rate of less than 1 percent per year has an output-per-worker level that is nearly 60 percent of the U.S. level. The average country with a labor-force growth rate of more than 3 percent per year has an output-per-worker level that is only 20 percent of the U.S. level. To some degree poor countries have fast labor-force growth rates because they are poor: causation runs both ways. Nevertheless, high labor-force growth rates are a powerful cause of low capital intensity and relative poverty in the world today.



Source: Author's calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, [www.nber.org](http://www.nber.org).

growth to affect average prosperity and living standards. But that it takes time is reason for governments that value their countries' long-run prosperity to take steps now (or even sooner) to start assisting the demographic transition to low levels of population growth. Female education, social changes that provide women with more opportunities than being a housewife, inexpensive birth control — all these pay large long-run dividends as far as national prosperity levels are concerned.

U.S. President John F. Kennedy used to tell a story of a retired French general, Marshal Lyautey, “who once asked his gardener to plant a tree. The gardener objected that the tree was slow-growing and would not reach maturity for a hundred years. The Marshal replied, ‘In that case, there is no time to lose, plant it this afternoon.’”

### The Saving Rate and the Price of Capital Goods

The most frequent sources of shifts in the parameters of the Solow growth model are shifts in the economy's saving-investment rate. The rise of politicians eager to promise goodies — whether new spending programs or tax cuts — to voters induces large government budget deficits, which can be a persistent drag on an economy's saving rate and its rate of capital accumulation. Foreigners become alternately overoptimistic and overpessimistic about the value of investing in our country, and

so either foreign saving adds to or foreign capital flight reduces our own saving-investment rate. And changes in households' fears of future economic disaster, in households' access to credit, or in any of numerous other factors change the share of household income that is saved and invested as well.

What effects do changes in saving rates have on the balanced-growth path levels of  $Y/L$ ? The higher the share of national product devoted to saving and gross investment — the higher is  $s$  — the higher will be the economy's balanced-growth capital-output ratio  $s/(n + g + \delta)$ . Why? Because more investment increases the amount of new capital that can be devoted to building up the average ratio of capital to output. Double the share of national product spent on gross investment, and you will find that you have doubled the economy's capital intensity, or its average ratio of capital to output.

One way to think about this is that the equilibrium is the point at which the economy's investment effort and its investment requirements are in balance. Investment effort is simply  $s$ , the share of total output devoted to saving and investment. Investment requirements are the amount of new capital needed to replace depreciated and worn-out machines and buildings, plus the amount needed to equip new workers who increase the labor force, plus the amount needed to keep the stock of tools and machines at the disposal of workers increasing at the same rate as the efficiency of their labor. So double the saving rate and you double the balanced-growth capital-output ratio, as seen in Box 4.8.

#### AN INCREASE IN THE SAVING-INVESTMENT RATE: AN EXAMPLE

To see how an increase in the economy's saving rate  $s$  changes the balanced-growth path for output per worker, consider an economy in which the parameter  $\alpha$  is  $2/3$ , the rate of labor-force growth  $n$  is 1 percent per year, the rate of labor efficiency growth  $g$  is 1.5 percent per year, and the depreciation rate  $\delta$  is 3.5 percent per year. Suppose that the saving rate  $s$ , which was 18 percent, suddenly and permanently jumped to 24 percent of output.

Before the increase in the saving rate, when  $s$  was 18 percent, the balanced-growth equilibrium capital-output ratio was

$$\frac{s}{n + g + \delta} = \frac{0.18}{0.01 + 0.015 + 0.035} = 3$$

After the increase in the saving rate, the new balanced-growth equilibrium capital-output ratio will be

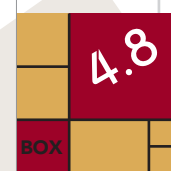
$$\frac{s}{n + g + \delta} = \frac{0.24}{0.01 + 0.015 + 0.035} = 4$$

Before the increase in saving, the balanced-growth path for output per worker was

$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}} (E_t) = (3)^{\frac{2/3}{1-2/3}} (E_t) = (3)^2 (E_t) = 9E_t$$

After the increase in saving, the balanced-growth path for output per worker will be


$$\frac{Y_t}{L_t} = \left( \frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}} (E_t) = (4)^{\frac{2/3}{1-2/3}} (E_t) = (4)^2 (E_t) = 16E_t$$



Divide the second equation by the first. We see that balanced-growth path output per worker after the jump in the saving rate is higher by a factor of 16/9, or fully 78 percent higher.

Just after the increase in saving has taken place, the economy is still on its old, balanced-growth path. But as decades and generations pass the economy converges to its new balanced-growth path, where output per worker is not 9 but 16 times the efficiency of labor. The jump in capital intensity makes an enormous difference for the economy's relative prosperity.

Note that this example has been constructed to make the effects of capital intensity on relative prosperity large: The high value for the diminishing-returns-to-investment parameter  $\alpha$  means that differences in capital intensity have large and powerful effects on output-per worker-levels.

But even here, the shift in saving and investment does not *permanently* raise the economy's growth rate. After the economy has settled onto its new balanced-growth path, the growth rate of output per worker returns to the same 1.5 percent per year that is  $g$ , the growth rate of the efficiency of labor. 

The same consequences as a low saving rate would follow from a country that makes the purchase of capital goods expensive. An abnormally high price of capital goods can translate a reasonably high saving effort into a remarkably low outcome in terms of actual gross additions to the real capital stock. The late economist Carlos Diaz-Alejandro placed the blame for much of Argentina's poor growth performance since World War II on trade policies that restricted imports and artificially boosted the price of capital goods. Economist Charles Jones reached the same conclusion for India. And economists Peter Klenow and Chang-Tai Hsieh argued that the world structure of prices that makes capital goods relatively expensive in poor countries plays a major role in blocking development.

How important is all this in the real world? Does a high rate of saving and investment play a role in making countries relatively rich not just in economists' models but in reality? It turns out that it is important indeed, as Figure 4.13 shows. Of the 22 countries in the world with output-per-worker levels at least half of the U.S. level, 19 have investment that is more than 20 percent of output. The high capital-output ratios generated by high investment efforts are a very powerful source of relative prosperity in the world today.

### Growth Rate of the Efficiency of Labor

By far the most important impact on an economy's balanced-growth path values of output per worker, however, is from shifts in the growth rate of the efficiency of labor  $g$ . We already know that growth in the efficiency of labor is absolutely essential for sustained growth in output per worker and that changes in  $g$  are the only things that cause permanent changes in growth rates that cumulate indefinitely.

Recall yet one more time the capital-output ratio form of the production function:

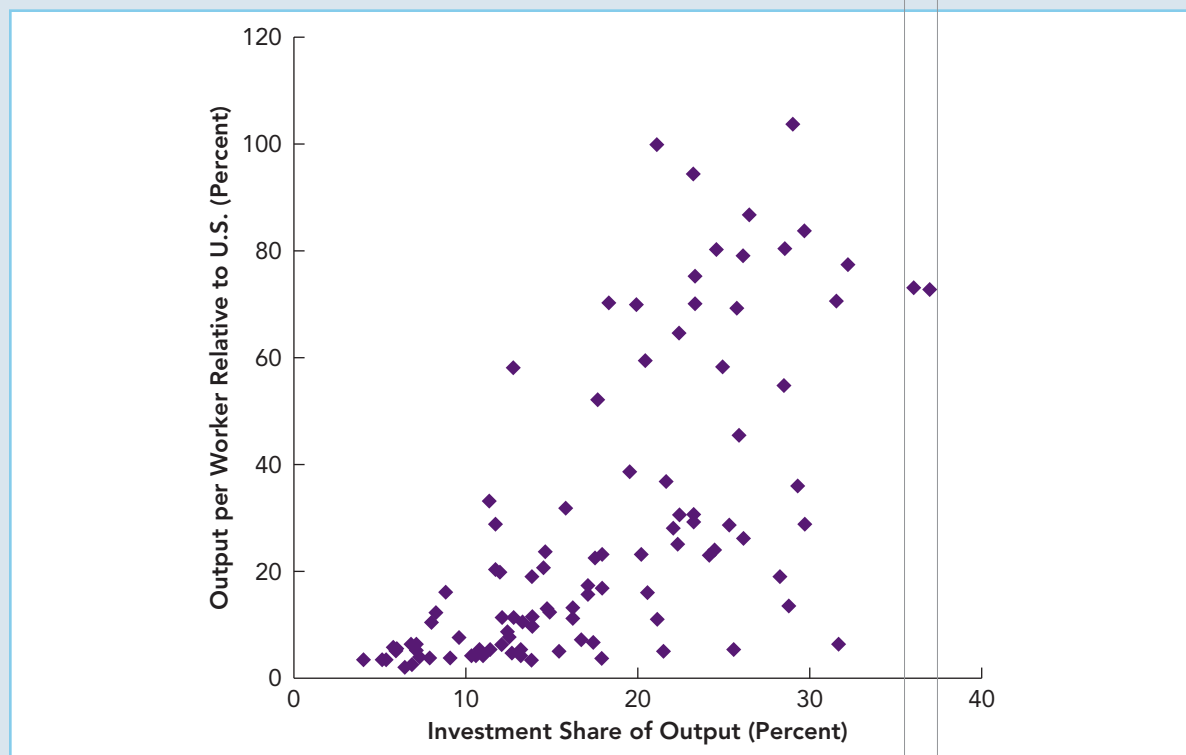
$$\frac{Y_t}{L_t} = \left( \frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} (E_t)$$

The capital-output ratio  $K/Y$  is constant along the balanced-growth path. The returns-to-investment parameter  $\alpha$  is constant. The balanced-growth path level of output per worker  $Y/L$  grows only if and only as fast as the efficiency of labor  $E$  grows.

**FIGURE 4.13**

**Investment Shares of Output and Relative Prosperity** The average country with an investment share of output of more than 25 percent has an output-per-worker level that is more than 70 percent of the U.S. level. The average country with an investment share of output of less than 15 percent has an output-per-worker level that is less than 15 percent of the U.S. level. This is not entirely due to a one-way relationship from a high investment effort to a high balanced-growth capital-output ratio: Countries are poor not just because they invest little; to some degree they invest little because they are poor. But much of the relationship is due to investment's effect on prosperity. High saving and investment rates are a very powerful cause of relative wealth in the world today.

Where is the U.S. on this graph? For these data it has an investment rate of 21 percent of GDP and an output-per-worker level equal (not surprisingly) to 100 percent of the U.S. level.



Source: Author's calculations from the Penn World Table data constructed by Alan Heston and Robert Summers, [www.nber.org](http://www.nber.org).

Increases or decreases in the rate of growth of the efficiency of labor  $g$  have effects on capital intensity that are in one sense just like changes in the rate of labor-force growth. An increase in the efficiency of labor growth rate  $g$  reduces the balanced-growth equilibrium capital-output ratio, just as an increase in labor-force growth did. And, as with a shift in labor-force growth, the level of output per worker after such a change in the labor efficiency growth rate begins the process of converging to its new balanced-growth path.

You might think that this means that an increase in  $g$  lowers output per worker  $Y/L$  — it lowers the capital-output ratio, after all. But you would be wrong. The effect on the capital-output ratio  $K/Y$  is only a small part of the story.

Changes in the efficiency of labor change the growth rate of output per worker along the balanced-growth path. In the very long run, no matter how large the

**TABLE 4.2**  
Effects of Increases in Parameters on the Solow Growth Model

When there is an increase in the parameter . . .	The effect on . . .				
	Equilibrium $K/Y$	Level of $Y$	Level of $Y/L$	Permanent Growth Rate of $Y$	Permanent Growth Rate of $Y/L$
$s$ saving-investment rate	Increases	Increases	Increases	No change	No change
$n$ labor-force growth rate	Decreases	Increases	Decreases	Increases	No change
$\delta$ depreciation rate	Decreases	Decreases	Decreases	No change	No change
$g$ efficiency of labor growth rate	Decreases	Increases	Increases	Increases	Increases

effects of a shift in efficiency of labor growth  $g$  on the economy's capital-output ratio, these effects are overwhelmed by the direct effect of  $g$  on output per worker. It is the economy with a high rate of efficiency of labor force growth  $g$  that becomes by far the richest over time.

This is our most important conclusion. The growth rate of the standard of living — of output per worker — can change if and only if the growth rate of labor efficiency changes. Other factors — a higher saving-investment rate, lower labor-force growth rate, or lower depreciation rate — can shift output per worker up as noted in Table 4.2, but they do not permanently change the growth rate of output per worker. Only a change in the growth rate of labor efficiency can permanently change the growth rate of output per worker. If we are to increase the rate of growth of the standard of living, we must pursue policies that increase the rate at which labor efficiency grows — policies that enhance technological or organizational progress. Chapter 5 looks at centuries of economic history for just those examples of those all important changes in the growth rate of labor efficiency.

#### RECAP USING THE SOLOW GROWTH MODEL

Changes in the economic environment and in economic policy can have powerful effects on the economy's long-run economic growth path. In the Solow model we analyze the effects of such changes by looking at their effects on capital intensity and on the efficiency of labor. Shifts in the growth rate of the efficiency of labor have the most powerful effects: They change the long-run growth rate of the economy. Shifts in other parameters affect the economy's capital intensity, affect what multiple of the efficiency of labor the balanced-growth path of output per worker follows, and make the economy richer or poorer as it converges to a new, different balanced-growth path. But only a change in the growth rate of labor efficiency can produce a permanent change in the growth rate of output per worker.

## Chapter Summary

1. One principal force driving long-run growth in output per worker is the set of improvements in the efficiency of labor springing from technological progress and advances in organization.
2. A second principal force driving long-run growth in output per worker is the increases in capital intensity, the ratio of the capital stock to output.
3. The balanced-growth equilibrium in the Solow growth model occurs when the capital output ratio  $K/Y$  is constant. When  $K/Y$  is constant, the capital stock and real output are growing at the same rate.
4. The Cobb-Douglas production function we use is

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha E^{(1-\alpha)}$$

This function is equivalent to

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} (E)$$

An increase in the returns-to-investment parameter  $\alpha$  makes the production function steeper. An increase in labor efficiency  $E$  makes the production function shift up.

5. In equilibrium, investment equals saving:  $I = S = S^H + S^G + S^F$ . We assume  $S/Y$ , the saving-investment rate  $s$ , is constant.
6. The balanced-growth equilibrium value of the capital output ratio  $K/Y$  is a constant equal to the saving rate  $s$  divided by the sum of the labor-force growth rate  $n$ , the labor efficiency growth rate  $g$ , and the depreciation rate  $\delta$ : in balanced-growth equilibrium,  $K/Y = s/(n + g + \delta)$ .
7. If the economy's actual value of  $K/Y$  is initially greater than  $s/(n + g + \delta)$ , then  $K/Y$  will fall until it reaches its equilibrium value. If the economy's actual value of  $K/Y$  is initially less than  $s/(n + g + \delta)$ , then  $K/Y$  will rise until it reaches its equilibrium value. It can take decades or generations for  $K/Y$  to reach its balanced-growth equilibrium value.
8. An increase in the saving rate  $s$ , a decrease in the labor-force growth rate  $n$ , or a decrease in the depreciation rate  $\delta$  increases output per worker  $Y/L$ . The growth rate of  $Y/L$  will accelerate as the economy moves to its new higher balanced-growth path. But once the economy is on its new balanced-growth path, output per worker will grow at the same rate as it did initially.
9. In balanced-growth equilibrium, the growth rate of output per worker equals the growth rate of labor efficiency  $g$ . So only an increase in the rate at which labor efficiency grows can produce a lasting increase in the rate of growth of output per worker.

## Key Terms

labor force (p. ●●●)

capital stock (p. ●●●)

efficiency of labor (p. ●●●)

capital intensity (p. ●●●)

balanced-growth path (p. ●●●)

production function (p. ●●●)

output per worker (p. ●●●)

saving rate (p. ●●●)

depreciation (p. ●●●)

capital-output ratio (p. ●●●)

balanced growth (p. ●●●)

balanced-growth equilibrium capital-output ratio (p. ●●●)

convergence (p. ●●●)

## Analytical Exercises

1. Consider an economy in which the depreciation rate is 3 percent per year, the rate of population increase is 1 percent per year, the rate of technological progress is 1 percent per year, and the private saving rate is 16 percent of GDP. Suppose that the government increases its budget deficit, which had been at 1 percent of GDP for a long time, to 3.5 percent of GDP and keeps it there indefinitely.
  - a. What will be the effect of this shift in policy on the economy's equilibrium capital-output ratio?
  - b. What will be the effect of this shift in policy on the economy's equilibrium balanced-growth path for

output per worker? How does your answer depend on the value of the diminishing-returns-to-investment parameter  $\alpha$ ?

- c. Suppose that your forecast of output per worker 20 years in the future had been \$100,000. What is your new forecast of output per worker 20 years hence?

2. Suppose that a country has the production function

$$Y = K^{0.5}(LE)^{0.5}$$

- a. Express output  $Y$  as a function of the level of the efficiency of labor  $E$ , the size of the labor force  $L$ , and the capital-output ratio  $K/Y$ .

- b. What is the expression for output per worker  $Y/L$ ?

3. Suppose that with the production function

$$Y = K^{0.5}(LE)^{0.5}$$

the depreciation rate on capital is 3 percent per year, the rate of population growth is 1 percent per year, and the rate of growth of the efficiency of labor is 1 percent per year.

- a. Suppose that the saving rate is 10 percent of GDP. What is the equilibrium capital-output ratio? What is the value of output per worker on the balanced-growth path written as a function of the level of the efficiency of labor?

- b. Suppose that the saving rate is 15 percent of GDP. What is the equilibrium capital-output ratio? What is the value of output per worker on the balanced-growth path?

- c. Suppose that the saving rate is 20 percent of GDP. What is the equilibrium capital-output ratio? What is the value of output per worker on the balanced-growth path?

4. What happens to the equilibrium capital-output ratio if the rate of technological progress increases? Would the balanced-growth path of output per worker for the economy shift upward, shift downward, or remain in the same position?

5. Discuss the following proposition: "An increase in the saving rate will increase the equilibrium capital-output ratio and so increase both output per worker and the rate of economic growth in both the short run and the long run."

6. Would the balanced-growth path of output per worker for the economy shift upward, shift downward, or remain the same if capital were to become more durable — if the rate of depreciation on capital were to fall?

7. Suppose that a sudden disaster — an epidemic, say — reduces a country's population and labor force but does not affect its capital stock. Suppose further that the economy was on its equilibrium balanced-growth path before the epidemic.

- a. What is the immediate effect of the epidemic on output per worker? On the total economywide level of output?

- b. What happens subsequently?

8. According to the marginal productivity theory of distribution, in a competitive economy the real rate of return on a dollar's worth of capital — its profits or interest — is equal to capital's marginal productivity. With the production function

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha E^{(1-\alpha)}$$

what is the marginal product of capital? That is, how much is total output ( $Y$ , not  $Y/L$ ) boosted by the addition of an extra unit to the capital stock?

9. According to the marginal productivity theory of distribution, in a competitive economy the real rate of return on a dollar's worth of capital — its profits or interest — is equal to capital's marginal productivity. If this theory holds and the marginal productivity of capital is indeed

$$\frac{dY}{dK} = \alpha \frac{Y}{K}$$

how large are the total earnings received by capital? What share of total output will be received by the owners of capital as their income?

10. Suppose that environmental regulations lead to a slowdown in the rate of growth of the efficiency of labor in the production function but also lead to better environmental quality. Should we think of this as a "slowdown" in economic growth or not?

## Policy Exercises

1. In the mid-1990s during the Clinton presidency the United States eliminated its federal budget deficit. The national saving rate was thus boosted by 4 percent of GDP, from 16 percent to 20 percent of real GDP. In the mid-1990s, the nation's rate of labor-force growth was 1 percent per year, the depreciation rate was 3 percent

per year, the rate of increase of the efficiency of labor was 1 percent per year, and the diminishing-returns-to-investment parameter  $\alpha$  was 1/3. Then when George W. Bush took office, the fiscal reforms of the Clinton administration were reversed, leading to deficits of 4 percent of GDP once again.

- a. Suppose that the federal budget deficit remains at 4 percent indefinitely. What will the U.S. economy's equilibrium capital-output ratio be? If the efficiency of labor in 2000 was \$30,000 per year, what would be your forecast of output per worker in 2040?
- b. Suppose that George W. Bush had not taken office, and that Clinton's successor Al Gore and his successors had continued to run a balanced budget. What would be your calculation of the U.S. economy's balanced-growth equilibrium capital-output ratio? If the efficiency of labor in 2000 was \$30,000 per year, what would be your forecast of output per worker in 2040?
2. How would your answers to question 1 change if your estimate of the diminishing-returns-to-investment parameter  $\alpha$  was not  $1/3$  but  $1/2$  and if your estimate of the efficiency of labor in 2000 was not \$30,000 but \$15,000 a year?
3. How would your answers to question 1 change if your estimate of the diminishing-returns-to-investment parameter  $\alpha$  was not  $1/3$  but  $2/3$ ?
4. What are the long-run costs as far as economic growth is concerned of a policy of taking money that could reduce the national debt — and thus add to national saving — and distributing it as tax cuts instead? What are the long-run benefits of such a policy? How can we decide whether such a policy is a good thing or not?
5. At the end of the 1990s it appeared that because of the computer revolution the rate of growth of the efficiency of labor in the United States had doubled, from 1 percent per year to 2 percent per year. Suppose this increase is permanent. And suppose the rate of labor-force growth remains constant at 1 percent per year, the depreciation rate remains constant at 3 percent per year, and the American saving rate (plus foreign capital invested in America) remains constant at 20 percent per year. Assume that the efficiency of labor in the United States in 2000 was \$15,000 per year and that the diminishing-returns-to-investment parameter  $\alpha$  was  $1/3$ .
- a. What is the change in the balanced-growth equilibrium capital-output ratio? What is the new capital-output ratio?
- b. Would such a permanent acceleration in the rate of growth of the efficiency of labor change your forecast of the level of output per worker in 2040?
6. How would your answers to question 5 change if your estimate of the diminishing-returns-to-investment parameter  $\alpha$  was not  $1/3$  but  $1/2$  and if your estimate of the efficiency of labor in 2000 was not \$30,000 but \$15,000 a year?
7. How would your answers to question 5 change if your estimate of the diminishing-returns-to-investment parameter  $\alpha$  was not  $1/3$  but  $2/3$ ?
8. Output per worker in Mexico in the year 2000 was about \$10,000 per year. Labor-force growth was 2.5 percent per year. The depreciation rate was 3 percent per year, the rate of growth of the efficiency of labor was 2.5 percent per year, and the saving rate was 16 percent of GDP. The diminishing-returns-to-investment parameter  $\alpha$  is 0.5.
- a. What is Mexico's equilibrium capital-output ratio?
- b. Suppose that Mexico today is on its balanced-growth path. What is the current level of the efficiency of labor  $E$ ?
- c. What is your forecast of output per worker in Mexico in 2040?
9. In the framework of question 8, how much does your forecast of output per worker in Mexico in 2040 increase if:
- a. Mexico's domestic saving rate remains unchanged but the nation is able to finance extra investment equal to 4 percent of GDP every year by borrowing from abroad?
- b. The labor-force growth rate immediately falls to 1 percent per year?
- c. Both *a* and *b* happen?
10. Consider an economy with a labor-force growth rate of 2 percent per year, a depreciation rate of 4 percent per year, a rate of growth of the efficiency of labor of 2 percent per year, and a saving rate of 16 percent of GDP. If the saving rate increases from 16 to 17 percent, what is the proportional increase in the equilibrium level of output per worker if the diminishing-returns-to-investment parameter  $\alpha$  is  $1/3$ ?  $1/2$ ?  $2/3$ ?  $3/4$ ?

